

Futures on Notional Bonds

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Government bond futures are contracts to buy forward bond issued by governments. Understanding and valuing these contracts is obscured by conventions used to quote bonds (the quoted price is not the paid price) and by the specifics of the definition of the notional contract (the underlying bond is virtual, it does not exist).

Let us start with an example.

We are September 9, 2005. The Euro-Bond Futures (FGBL) contract for December 2005 trading on Eurex is quoted as 123.09. What does this quotation mean?

When buying a contract (by being long on a contract), we will be buying on the 24st of December 2005 a bond issued by the Government of Federal Germany (Bund), with a maturity in between 8.5 and 10.5 years and with a nominal value of EUR 100,000. What will we receive at maturity? What is the delivery price fixed today?

The quoted price takes into account the difference between the coupon of the notional bond (6% for a Bund contract) and the coupon of the physical bond that will be delivered to the buyer at the maturity of the contract.

Let us suppose that the physical bond that will be delivered is the DE0001135259 (ISIN code). We will refer to it as the Bund 259. The coupon rate is 4.25% and the maturity date 04/07/2014. Let us also assume that this bond is quoted at 109.55¹ on the 9th of September 2005. The accrued interest since the last detachment of a coupon is 0.780². Consequently, the cash price is equal to $109.55 + 0.780 = 110.33$. The conversion factor associated with this bond for maturity in December 2005 is 0.885160 (www.eurexchange.com). At the end on December 2005, the accrued interest for the Bund 259 will be equal to 2.014.

Taking a long position on an FGBL contract is equivalent to buying Bund 259 forward on the 31st of December 2005. The delivery price that is set today is equal to the quoted futures price multiplied by the conversion factor to which one must add accrued interest. The cash price we will pay is thus be equal to:

$$123.09 \times 0.885160 + 2.014 = 110.97$$

The quoted futures price is multiplied by the conversion factor to take into account the difference between the coupon of the notional contract (6%) and the coupon of the physical bond (4.25%). As the conversion factor is inferior to one, the adjustment in the quoted futures price results in the lowering of the delivery price (the “clean” delivery price is equal to $123.09 \times 0.885160 = 108.95$) of the physical bond, thereby taking into account the fact that the physical bond’s coupon is less than that of the notional bond’s.

¹ Own calculations (using PRICE function in EXCEL) under the hypothesis that the yield to maturity is 3% and that the quotation is made on the basis of Actual/365. (See Annex)

² Own calculations (using COUPDAYBS function), Actual/365 basis.

As a contract has a nominal value of EUR 100,000, the amount that we will pay is EUR 110,970. In exchange, we will receive the Bund 259 with a nominal value of EUR 100,000. The profit or loss resulting from the operation will be equal to the difference between the value of the bond that we will receive and the price that we will pay.

If, for example, the Bund 259 is quoted at 110 on the 31st of December, the Bund that will be delivered will have a cash price of $110 + 2.014 = 112.014$ and a market value of EUR 112,014. We will therefore have made a profit of $112,014 - 110,970 = 1,047$ EUR.

Notice that the accrued interest does not play a role for determining the profit or loss of this operation.

We can obtain the result directly by performing the following calculation:

$$100,000 \times (110 - 123.09 \times 0.885160) / 100 = 1,047 \text{ EUR}$$

For those who prefer a formula:

$$\pi_{T\text{forw}} = N \times (s_T - F_t \times k) / 100$$

where N is the size of the contract, s_T the quoted price of the underlying physical bond at maturity, F_t the quoted futures price at date t and k the conversion factor.

(I had promised myself not to overload you with formulas but this one is irresistible and will be useful later on.)

The previous analysis has assimilated the futures to a forward contract. Let us now examine the consequences of daily profit and loss settlement (marking to market).

Let us go back to the previous example of a long position on a futures with a price of 123.09. For reasons that, I hope, will become clearer later on, the number of futures contracts necessary to obtain the same protection as from a forward is equal to the conversion factor of the underlying physical bond. In our example, we will therefore buy 0.885160 futures contracts.

Every day, the buyer's account will be debited or credited by the amount resulting from his position during the day. For a long position, this result is equal to the variation of the futures price, multiplied by the size of the contract. For the Bund contract quoted on Eurex, the size of the contract is EUR 100,000. Thus, if the futures price grew from 123.09 to 123.59, the buyer would earn, per contract:

$$100,000 \times (123.59 - 123.09) / 100 = 500 \text{ EUR}$$

The tick value (value of one basis point, i.e. 0.01%) is equal to €10 in this case ($100,000 \times 0.01/100$). An increase of 50 basis points in the quoted futures price would therefore mean a result per contract of:

$$50 \text{ bp} \times 10 \text{ €/bp} = 500 \text{ EUR}$$

The sum of daily variations in the quoted futures price between the date on which the contract comes to maturity (T) and the date on which the contract was bought (t) is equal to the difference between the futures price at maturity and the futures price on the date when bought. The result over the period when the contract was held can thus be calculated by multiplying this difference (expressed in basis points) by the tick.

Let us suppose, as before, that the Bund 259 is quoted at 110 at maturity. As the futures price at maturity converges towards the price of the underlying bond, the futures price at maturity will therefore be the solution of:

$$0.885160 \times F_T = 110$$

i.e.

$$F_T = 124.27$$

Let us calculate the result for the buyer in between the buying date and the date of maturity of the contract. The variation in the futures price is 118 basis points $(124.27 - 123.09) \times 100$. The result is thus equal to $118 \times 10 = 1,180$ EUR per contract.

We notice here that the result per futures contract (EUR 1,180) is different from the result obtained previously when we assimilated the futures to a forward (EUR 1,047). A formula will help us to understand this.

As a reminder, the payoff on a forward contract is:

$$\pi_{T,\text{forw}} = N \times (s_T - F_t \times k) / 100$$

On the other hand, the result on a futures contract is given by:

$$\pi_{T,\text{fut}} = N \times (F_T - F_t) / 100$$

As:

$$s_T = F_T \times k$$

the result on the futures contract can be written as:

$$\pi_{T,\text{fut}} = [N \times (s_T - F_t \times k) / 100] / k = \pi_{T,\text{forw}} / k$$

This is the result on the forward contract divided by the conversion factor. This explains why a long position on k contracts leads to the same result as on one forward contract.

This result is a special case of the formula to determine the number of future contracts needed to be bought in order to cover a bond portfolio:

$$n = \frac{M}{N} \times \frac{D_{Targ}}{D_{ctd}} \times \frac{S}{s} \times k$$

where M is the nominal portfolio value, N the size of the contract, D_{Targ} the duration of the portfolio, D_{ctd} the duration of the cheapest-to-deliver bond, S is the average price of the bonds in the portfolio ($M \times S$ is the market value of the bond portfolio) s the price of the cheapest-to-deliver bond and k the conversion factor of the cheapest-to-deliver bond.

To be able to see this clearly, suppose a financial intermediary plans to buy Bund 259 for a nominal value of EUR 100,000 in December 2005. She wants to cover her position with futures.

By applying the formula she would have obtained $n = k$ as $M = N$, $D_{Targ} = D_{ctd}$ and $S = s$.

On delivery, the buyer pays to the seller the price at maturity (124.27) multiplied by the conversion factor (0.885160) in order to take into account the characteristics of the physical bond i.e. 110 (he pays the spot price quoted at maturity) to which he must add accrued interest. She will therefore pay:

$$100,000 \times (110.97\% + 2.014\%) = 112,014 \text{ EUR}$$

However, he will have made a profit of $0.885160 \times 1,180 = 1,046$ on her futures position which brings total acquisition costs to $112,014 - 1,046 = 110,969$, which is, providing for rounding, the same result as obtained for the forward contract (110,970).

Let us proceed further in our analysis. Up until now, we have assumed that the physical bond was known. In reality, the seller can choose which bond to deliver amongst a list of bonds (a pool). He will choose the bond that is the cheapest to deliver and it is that bond that becomes the underlying asset of the contract. This brings us to two questions:

1. which bond is cheapest-to-deliver?
2. given the price of this bond, what should be the quoted futures price?

The cheapest-to-deliver bond is the bond that maximizes the difference $k_j F_t - s_j$ which is basically equivalent to the difference between what the seller will receive at maturity and the value of the physical bond that he will deliver. In our case, there are 3 deliverable bonds: Bunds 259, 267, 283. Table 1 calculates this difference for each of these bonds. One can see that Bund 259 would be the cheapest-to-deliver if the given data were correct³. Table 2 calculates the theoretical forward price in 4 steps:

1. cash price calculation (quoted price + accrued interest)
2. forward cash price calculation (future value of the difference between the cash price and the present value of the coupon that would be obtained before maturity of the option)
3. quoted forward price calculation (forward cash price – accrued interest at maturity)
4. quoted futures price calculation (quoted forward price divided by conversion factor)

The difference of -0.05 between market price (123.09) and the calculated price (123.14) results from the rounding of prices, accrued interest and interest rates. I will try and do better next time.

³ In reality, not having the quoted prices of the bonds in the pool, I calculated them based on the 10 year rate. The incurred interest (used for switching between dirty and clean prices) is also my own estimation.

Table 1 Determining the cheapest-to-deliver bond

Euro-Bund Futures

Date: 9 septembre 2005

Futures maturity: December 2005

Quoted price: 123,09

Deliverable bond	Coupon	Quoted spot price (clean price)	Conversion factor	$k_j F_t - s_j$
Bund 259	4.25	109.55	0.885160	-0.05
Bund 267	3.75	106.01	0.846069	-1.71
Bund 283	3.25	102.09	0.803899	-3.52

Table 2 Calculation of Euro-Bond Futures price December 2005

Cheapest-to-deliver bond = Bund 259

Interest rate 3 months = 2.13% (simple interest)

Quoted spot price	109.55
Accrued interest	0.78
Cash spot price	110.33
Cash futures price	111.01
- Accrued interest	2.014
= Quoted forward price	109.00
/ Conversion factor	0.885160
= Quoted futures price	123.14

Annex Accrued Interest Calculation

Accrued interest (AccruedInt) calculation is done by multiplying the coupon by the ratio of the number of days between the last coupon payment (n) and the number of days in a year (a):

$$\text{AccruedInt} = \text{Coupon} \times n/a$$

Even though the principle is simple, its application depends on conventions used to calculate n and a .

For n , two methods are used:

- n is the exact number of days, taking into account the different number of days in each month
- n is calculated assuming every month has 30 days

For the number of days per year, there are three methods:

- the actual number of days (actual)
- 360 days per year (360)
- 365 days per year (365)

By combining these criteria, the basis for the calculation of AccruedInt can be one of the following:

30/360

Actual/actual

Actual/360

Actual/365