Théorie Financière

Structure financière et coût du capital

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Risk and return and capital budgeting

- Objectives for this session:
  - Beta of a portfolio
  - Beta and leverage
  - Weighted average cost of capital
  - Modigliani Miller 1958
  - WACC with taxes
Levers of Accounting Performance

- Return on Equity
  - Return on Invested Capital
    - Profit Margin
    - Asset Turnover
  - Leverage
Return on invested capital

- Return on assets (net) = Net income / Total assets
- Advantage: fits with DuPont system
  - ROE = ROA x Equity multiplier
- Limitation: Net income = EBIT - Interest expense - Taxes
  - Depends on capital structure:
    - 1. Interest expense: function of interest-bearing debt
    - 2. Interest expense: tax deductible
- Preferred measure: Return on Invested Capital (ROIC)

$$\text{ROIC} = \frac{\text{EBIT}(1 - \text{TaxRate})}{\text{Stockholders' equity} + \text{Interest bearing debt}}$$

- NB: ROIC = ROA (gross) (1 - Tax rate)
- = ROE of a all equity financed firm
Financial leverage

• Financial leverage magnifies ROE only when ROA (gross) is greater than the interest rate on debt.
• Balance sheet: \( TA = SE + D \)
• Income statement: \( NI = EBIT - INT - TAX \)
• Interest expense \( INT = r \cdot D \) (Interest expense = Interest rate \( r \) x Interest-bearing debt)
• Taxes \( TAX = (EBIT - r \cdot D) \cdot T_c \) (Taxes = Taxable income \( EBIT - INT - TAX \) x Tax rate \( T_c \))

\[
ROE = \frac{NI}{SE} = \frac{EBIT \times (1 - T_c)}{TA} \times \frac{TA}{SE} - r(1 - T_c) \times \frac{D}{SE}
\]

\[
ROE = ROIC + (ROIC - r(1 - T_c)) \times \frac{D}{SE}
\]

• Remember: \( ROIC = ROA_{gross} \times (1 - T_c) \)
  • \( ROE = ROIC + (ROA_{gross} - r) (1 - T_c) \times (D/SE) \)
Leverage - example

Cost of debt 4% 4%
Tax Rate 30% 30%

Balance sheet
Total asset 100,000 100,000
Book Equity 100,000 5,000
Debt 0 95,000

Income Statement
EBIT 5,500 5,500
Interest 0 3,800
Taxes 1,650 510
Net Income 3,850 1,190

Return on Equity = 3.85% 23.80%
Return on Invested Capital ROIC 3.85% 3.85%
+ [ROIC - rD(1-Tc)] 1.05% 1.05%
X Debt / Book Equity 0.00% 19.00

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Beta of a portfolio

- Consider the following portfolio

<table>
<thead>
<tr>
<th>Stock</th>
<th>Value</th>
<th>Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>$n_A \times P_A$</td>
<td>$\beta_A$</td>
</tr>
<tr>
<td>B</td>
<td>$n_B \times P_B$</td>
<td>$\beta_B$</td>
</tr>
</tbody>
</table>

- The value of the portfolio is:

$$V = n_A \times P_A + n_A \times P_B$$

- The fractions invested in each stock are:

$$X_i = \frac{(n_i \times P_i)}{V} \quad \text{for } i = A, B$$

- The beta of the portfolio is the weighted average of the betas of the individual stocks

$$\beta_P = X_A \beta_A + X_B \beta_B$$
### Example

<table>
<thead>
<tr>
<th>Stock</th>
<th>$</th>
<th>$β</th>
<th>$X</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATT</td>
<td>3,000</td>
<td>0.76</td>
<td>0.60</td>
</tr>
<tr>
<td>Genetech</td>
<td>2,000</td>
<td>1.40</td>
<td>0.40</td>
</tr>
<tr>
<td>$V$</td>
<td>5,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ β_p = 0.60 \times 0.76 + 0.40 \times 1.40 = 1.02 \]
Application 1: cost of capital for a division

- Firm = collection of assets
- Example: A company has two divisions

<table>
<thead>
<tr>
<th></th>
<th>Value($ mio)</th>
<th>β</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electrical</td>
<td>100</td>
<td>0.50</td>
</tr>
<tr>
<td>Chemical</td>
<td>500</td>
<td>0.90</td>
</tr>
<tr>
<td>V</td>
<td>600</td>
<td></td>
</tr>
</tbody>
</table>

\[ \beta_{\text{firm}} = \left(\frac{100}{600}\right) \times 0.50 + \left(\frac{500}{600}\right) \times 0.90 = 0.83 \]

- Assume: \( r_f = 5\% \quad r_M - r_f = 6\% \)
- Expected return on stocks: \( r = 5\% + 6\% \times 0.83 = 9.98 \% \)
- An adequate hurdle rate for capital budgeting decisions? No
- The firm should use required rate of returns based on project risks:
  - Electricity: \( 5 + 6 \times 0.50 = 8\% \)
  - Chemical: \( 5 + 6 \times 0.90 = 10.4\% \)
Application 2: leverage and beta

- Consider an investor who borrows at the risk free rate to invest in the market portfolio
- Assets $ $ β X
- Market portfolio 2,000 1 2
- Risk-free rate -1,000 0 -1
- V 1,000

- $ \beta_p = 2 \times 1 + (-1) \times 0 = 2$
Expected Return vs. Sigma

- 14% at point M
- 8% at point P

Expected Return vs. Beta

- 20% at point P
- 14% at point M
Cost of capital with debt

• Up to now, the analysis has proceeded based on the assumption that investment decisions are independent of financing decisions.

• Does
  • the value of a company change
  • the cost of capital change
• if leverage changes?
An example

- CAPM holds – Risk-free rate = 5%, Market risk premium = 6%
- Consider an all-equity firm:
  - Market value $V = 100$
  - Beta = 1
  - Cost of capital = 11% ($=5\% + 6\% \times 1$)
- Now consider borrowing 20 to buy back shares.
- Why such a move?
  - Debt is cheaper than equity
  - Replacing equity with debt should reduce the average cost of financing
- What will be the final impact
  - On the value of the company? (Equity + Debt)?
  - On the weighted average cost of capital (WACC)?
Weighted Average Cost of Capital

- An average of:
  - The cost of equity $r_{equity}$
  - The cost of debt $r_{debt}$
  - Weighted by their relative market values ($E/V$ and $D/V$)

\[ r_{wacc} \equiv r_{equity} \times \frac{E}{V} + r_{debt} \times \frac{D}{V} \]

- Note: $V = E + D$
Modigliani Miller (1958)

- Assume perfect capital markets: not taxes, no transaction costs

- Proposition I:
  - The market value of any firm is independent of its capital structure:
    \[ V = E + D = V_U \]

- Proposition II:
  - The weighted average cost of capital is independent of its capital structure
    \[ r_{wacc} = r_A \]
  - \( r_A \) is the cost of capital of an all equity firm
Using MM 58

- Value of company: \( V = 100 \)

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>Final</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity</td>
<td>100</td>
<td>80</td>
</tr>
<tr>
<td>Debt</td>
<td>0</td>
<td>20</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>100</td>
</tr>
</tbody>
</table>

- WACC = \( r_A \) 11% 11% MM II

- Cost of debt - 5% (assuming risk-free debt)
- \( D/V \) 0 0.20
- Cost of equity 11% 12.50% (to obtain \( r_{wacc} = 11\% \))
- \( E/V \) 100% 80%
Cost of equity calculation

\[ V (=V_U) = E + D \]

Value of all-equity firm

Value of equity

Value of debt

\[ r_A = r_E \frac{E}{V_L} + r_D \frac{D}{V_L} \]
Why is $r_{wacc}$ unchanged?

- Consider someone owning a portfolio of all firm’s securities (debt and equity) with $X_{equity} = E/V$ (80% in example) and $X_{debt} = D/V$ (20%)

- Expected return on portfolio = $r_{equity} * X_{equity} + r_{debt} * X_{debt}$
- This is equal to the WACC (see definition):
  
  $$r_{portfolio} = r_{wacc}$$

- But she/he would, in fact, own a fraction of the company. The expected return would be equal to the expected return of the unlevered (all equity) firm

  $$r_{portfolio} = r_A$$

- The weighted average cost of capital is thus equal to the cost of capital of an all equity firm

  $$r_{wacc} = r_A$$
What are MM I and MM II related?

- Assumption: perpetuities (to simplify the presentation)
- For a levered companies, earnings before interest and taxes will be split between interest payments and dividends payments
  \[ EBIT = Int + Div \]
- Market value of equity: present value of future dividends discounted at the cost of equity
  \[ E = \frac{Div}{r_{equity}} \]
- Market value of debt: present value of future interest discounted at the cost of debt
  \[ D = \frac{Int}{r_{debt}} \]
Relationship between the value of company and WACC

- From the definition of the WACC:
  \[ r_{wacc} \times V = r_{equity} \times E + r_{debt} \times D \]

- As
  \[ r_{equity} \times E = Div \quad \text{and} \quad r_{debt} \times D = Int \]
  \[ r_{wacc} \times V = EBIT \]

Market value of levered firm
EBIT is independent of leverage
If value of company varies with leverage, so does WACC in opposite direction
• The equality $r_{\text{wacc}} = r_A$ can be written as:

\[
 r_{\text{equity}} = r_A + (r_A - r_{\text{debt}}) \times \frac{D}{E}
\]

• Expected return on equity is an increasing function of leverage:

![Graph showing the relationship between $r_{\text{equity}}$, $r_A$, $r_{\text{debt}}$, $D/E$, and additional cost due to leverage.]}
Why does $r_{equity}$ increases with leverage?

- Because leverage increases the risk of equity.
- To see this, back to the portfolio with both debt and equity.

- Beta of portfolio: $\beta_{portfolio} = \beta_{equity} \times X_{equity} + \beta_{debt} \times X_{debt}$
- But also: $\beta_{portfolio} = \beta_{Asset}$
- So:

$$\beta_{Asset} = \beta_{Equity} \times \frac{E}{E + D} + \beta_{Debt} \times \frac{D}{E + D}$$

- or

$$\beta_{Equity} = \beta_{Asset} + (\beta_{Asset} - \beta_{Debt}) \times \frac{D}{E}$$
• Assume debt is riskless:

\[ \beta_{\text{Equity}} = \beta_{\text{Asset}} \left(1 + \frac{D}{E}\right) = \beta_{\text{Asset}} \frac{V}{E} \]

\[ \beta_{\text{Equity}} = 1 \times \left(1 + \frac{20}{80}\right) = 1.25 \]

\[ r_{\text{Equity}} = r_F + (r_M - r_F) \beta_{\text{Equity}} = 5\% + 6\% \times 1.25 = 12.50\% \]
Summary: the Beta-CAPM diagram

\[ r = r_F + (r_M - r_F) \beta \]

\[ \beta_{\text{Equity}} = \beta_{\text{Asset}} + \beta_{\text{Asset}} \frac{D}{E} \]

\[ r_{\text{Equity}} = r_{\text{Asset}} + (r_{\text{Asset}} - r_{\text{Debt}}) \frac{D}{E} \]

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Risky debt: Merton model

- Limited liability: rules out negative equity

![Graph showing equity similar to a call option on the company]

- Market value of equity
- Equity similar to a call option on the company
- Company goes bankrupt if $V < F$
- Market value of company $V$
- Face value $F$

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Risk debt decomposition

Risky debt = Risk-free debt – Put

\[ 80 = 100 - 20 \]
The blue curve is the beta of equity and the red curve is the beta of debt as a function of the firm’s debt-to-equity ratio. The black line is the approximation derived in Chapter 14—the beta of equity when the beta of debt is assumed to be zero. The firm is assumed to hold five-year zero-coupon debt and reinvest all its earnings. (The firm’s beta of assets is 1, the risk-free interest rate is 3% per year, and the volatility of assets is 20% per year.)

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Corporate Tax Shield

- Interest are tax deductible => tax shield
- Tax shield = Interest payment × Corporate Tax Rate
  = \( (r_D \times D) \times T_C \)
- \( r_D \): cost of new debt
- \( D \): market value of debt
- Value of levered firm
  = Value if all-equity-financed + PV(Tax Shield)
- PV(Tax Shield) - Assume permanent borrowing

\[
PV(TaxShield) = \frac{T_C \times r_D D}{r_D} = T_C D
\]

\[
V_L = V_U + T_C D
\]
Cost of equity calculation

\[ V_U + T = E + D \]

Value of all-equity firm: \( r_A \)

Value of tax shield: \( r_D \)

Value of equity: \( r_E \)

Value of debt: \( r_D \)

\[
r_A \frac{V_U}{V_L} + r_D \frac{T_C D}{V_L} = r_E \frac{E}{V_L} + r_D \frac{D}{V_L}
\]
Cost of equity calculation (2)

General formula

\[
\begin{align*}
    r_A \frac{V_L - VTS}{V_L} + r_{TS} \frac{VTS}{V_L} &= r_A \frac{E}{V_L} + r_D \frac{D}{V_L} \\
    r_A (V_L - VTS) + r_{TS} VTS &= r_A E + r_D D \\
    r_E &= r_A + (r_A - r_D) \frac{D}{E} - (r_A - r_{TS}) \frac{VTS}{E}
\end{align*}
\]

If \( r_{TS} = r_D \) \& \( VTS = TCD \)

\[
    r_E = r_A + (r_A - r_D)(1 - T_C) \frac{D}{E}
\]

\[
    \beta_E = \beta_A + (\beta_A - \beta_D)(1 - T_C) \frac{D}{E}
\]

Similar formulas for beta equity (replace \( r \) by \( \beta \))
WACC

\[ wacc = r_E \frac{E}{V_L} + r_D (1 - T_C) \frac{D}{V_L} \]

\[ = r_A \frac{V_U}{V_L} < r_A \]
Assume $r_A = 10\%$

### Balance Sheet

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total Assets</td>
<td>1,000</td>
<td>1,000</td>
</tr>
<tr>
<td>Book Equity</td>
<td>1,000</td>
<td>500</td>
</tr>
<tr>
<td>Debt (8%)</td>
<td>0</td>
<td>500</td>
</tr>
</tbody>
</table>

(1) Value of all-equity-firm:

$V_U = \frac{144}{0.10} = 1,440$

(2) PV(Tax Shield):

Tax Shield $= 40 \times 0.40 = 16$

$PV(\text{Tax Shield}) = \frac{16}{0.08} = 200$

### Income Statement

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>EBIT</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>Interest</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>Taxable Income</td>
<td>240</td>
<td>200</td>
</tr>
<tr>
<td>Taxes (40%)</td>
<td>96</td>
<td>80</td>
</tr>
<tr>
<td>Net Income</td>
<td>144</td>
<td>120</td>
</tr>
<tr>
<td>Dividend</td>
<td>144</td>
<td>120</td>
</tr>
<tr>
<td>Interest</td>
<td>0</td>
<td>40</td>
</tr>
<tr>
<td>Total</td>
<td>144</td>
<td>160</td>
</tr>
</tbody>
</table>

(3) Value of levered company:

$V_L = 1,440 + 200 = 1,640$

(5) Market value of equity:

$E_L = V_L - D = 1,640 - 500 = 1,140$
What about cost of equity?

1) Cost of equity increases with leverage:

\[ r_E = r_A + (r_A - r_D) \times (1 - T_C) \times \frac{D}{E} \]

2) Beta of equity increases

\[ \beta_E = \beta_A \left[ 1 + (1 - T_C) \frac{D}{E} \right] \]

Proof:

\[ E = \frac{(EBIT - r_D D) \times (1 - T_C)}{r_E} \]

But \( V_U = EBIT(1-T_C)/r_A \)

and \( E = V_U + T_C D - D \)

Replace and solve

In example:

\[ r_E = 10\% + (10\%-8\%)(1-0.4)(500/1,140) = 10.53\% \]

or

\[ r_E = DIV/E = 120/1,140 = 10.53\% \]
What about the weighted average cost of capital?

- Weighted average cost of capital decreases with leverage
- *Weighted average cost of capital:* discount rate used to calculate the market value of firm by discounting net operating profit less adjusted taxes (NOPLAT)
- \( \text{NOPLAT} = \text{Net Income} + \text{Interest} + \text{Tax Shield} \)
  \[ \text{NOPLAT} = (\text{EBIT} - r_D D)(1-T_C) + r_D D + T_C r_D D \]
  \[ = \text{Net Income for all-equity-firm} = \text{EBIT}(1-T_C) \]
  \[ V_L = \frac{\text{NOPLAT}}{\text{WACC}} \]
- As:
  \[ r_E E + r_D (1 - T_C) D = \text{EBIT} (1 - T_C) \]
  \[ \text{WACC} = r_E \times \frac{E}{V_L} + r_D (1 - T_C) \times \frac{D}{V_L} \]

In example:
\( \text{NOPLAT} = 144 \)
\( V_L = 1,640 \)
\( \text{WACC} = 10.53\% \times 0.69 + 8\% \times 0.60 \times 0.31 = 8.78\% \)