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The Real-World Pitfalls of Portfolio Insurance

Mark Rubinstein, the Paul Stephens professor of applied investment analysis at the University of California at Berkeley’s Haas School of Business, discusses problems that arise in implementing portfolio insurance and what can be done about them.

Portfolio insurance is a popular application for dynamic hedging. It appeals rationally to investors who feel strongly, relative to other investors, that they cannot tolerate losses at all—but nonetheless would like to invest in risky assets because they are attracted by their high expected returns.

The first priority in managing an “insured portfolio” is to ensure that its value does not end up below some minimum level on a prespecified target payoff date. This minimum level is commonly termed the floor. Compared with investing in the underlying portfolio, on the downside an insured portfolio loses nothing. Because an insured portfolio should perform better than the underlying portfolio on the downside, it must perform worse on the upside (otherwise it would be a free lunch). Given the floor on the downside, the second priority is to minimize this shortfall relative to the performance of the underlying portfolio on the upside. Traditionally, the upside shortfall is measured by the upside capture, the ratio of the value of the insured portfolio to the price of the underlying portfolio on the payoff date.

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Despite its name, the insured portfolio in practice is not typically literally insured (guaranteed by a second party). Rather, if everything works out as anticipated, the results should be almost indistinguishable from a portfolio that is actually insured. Although portfolio insurance can sometimes be implemented with listed options, this discussion will focus on its implementation by dynamic hedging using only the underlying portfolio (or, more commonly, a highly correlated portfolio of futures) and cash. The basic feature of the dynamic hedging strategy is selling out of the underlying portfolio as its price falls, and buying more of the underlying portfolio as its price rises. The former implements a floor on losses and the latter ensures the upside capture. A common way to determine exactly how much to be selling or buying over time is to use a modified version of the Black-Scholes formula and its derived delta. In practice, several problems with portfolio insurance implementations can arise that are not anticipated by the Black-Scholes formula—and these are the chief concern of this article.

If Treasury bills are used as the proxy for cash, their interest rate is known with certainty over their life. However, the rate of change of the T-bill price over each day is still uncertain. For the most part, this can be handled by using as cash a zero-coupon bond maturing near the payoff date of the strategy. In addition, the upside capture (and delta) can be recalculated each day based on the remaining riskless return through the payoff date. This has the added virtue of preserving the floor even in the presence of uncertain future spot returns.

Since this kind of uncertainty is usually not significant, it has little effect on the outcome of the replicating strategy and further adjustments can be ignored. But in more refined implementations where greater accuracy is required, it can be considered more precisely by using a generalization of the Black-Scholes formula that allows for uncertain future spot returns.

A more difficult problem is that the volatility of the underlying portfolio is not known in advance. Several steps can be taken to reduce the seriousness of this situation. First, calculate the predicted upside capture considering the realized volatility so that the investor knows to expect less (more) upside capture after periods of high (low) volatility. Second, recalculate the upside capture (and delta) each day based on the current value of the insured portfolio—this preserves the floor even in the presence of uncertain volatility. Third, base the new calculation as well on a revised estimate of volatility, which is a mixture of the realized volatility so far and the anticipated volatility, putting less weight on anticipated volatility as the payoff date approaches. Fourth, if you think you
know how volatility is likely to change in the future, in place of the Black-Scholes formula use an option-pricing model that captures the volatility changes you anticipate.

Problems created by uncertain riskless returns and uncertain volatility can be reduced by combining static positions in options with dynamic replication. Indeed, if the replicating strategy could be achieved using only buy-and-hold positions in European exchange-traded options (static replication), then uncertain riskless returns and uncertain volatility would not be a problem at all. Jumps in underlying portfolio prices would also not pose a problem. In fact, any dependence of the results on the special assumptions behind the Black-Scholes formula would be removed.

Figure 1 shows how the predicted upside outcome of portfolio insurance depends on the assumed underlying portfolio volatility. The slope of the payoff shifts from zero to the upside capture at the point along the horizontal axis, which equals the floor divided by the upside capture.

At higher levels of volatility, the upside capture falls. However, recalculating the upside capture before each trade using the current value of the insured portfolio (and new estimates of the remaining riskless return and volatility) and then using this revised upside capture to calculate delta makes the floor insensitive to volatility, as indicated in Figure 1.

Following this procedure, it is as if the portfolio insurance strategy were reinitiated before each trade using the current value of the insured portfolio as the underlying portfolio value, with whatever time remains until the payoff date.

Only (unexpected) jumps in the underlying portfolio price that occur at “inopportune” (usually high gamma) times can derail the replicating strategy below the floor. Using the procedure discussed earlier, uncertain volatility, by itself, cannot push the strategy through the floor.

If the basic portfolio insurance strategy is followed scrupulously, it can lead to extremely high trading costs near the payoff date.

One way to prevent the insured portfolio from going though the floor would be to make the following calculation each day. Ask a question: At the prevailing riskless return, if the current insured portfolio were entirely invested in cash, would the floor be surpassed by the payoff date? As long as the answer to that question is yes, continue to follow the dynamic replication strategy with positive upside capture. At the first instance when the answer is no, positive upside capture (while guaranteeing the floor) is not possible. One should immediately become fully invested in cash, and stay invested in cash from that point through the payoff date. This will be the only way to be assured of coming close to or hitting the floor. Essentially, our bridges have been burned behind us, and we cannot afford to take any more risk without further jeopardizing the floor.

This is called the stop-out point and is demonstrated in Figure 2. The jagged lines are two possible sample paths of the value of the insured portfolio. The straight diagonal line is the lowest value to which the insured portfolio can fall without being forced completely into cash to deliver the floor. The line starts out low because with more time remaining until the payoff date, to make up the loss, more interest can be earned to reach the floor by the payoff date. The graph shows the lower jagged line hitting the minimum value line. At this stop-out point, to preserve the floor the replicating portfolio is switched completely into cash. From this point, simply by earning interest it travels up the minimum value line to the floor on the payoff date.

One hopes this point will never be reached. Indeed, as long as we revise to new deltas relatively frequently, volatility and riskless returns turn out as predicted, and there are no serious jumps in the underlying portfolio value, all will be well. However, things do happen; and to assure the floor, a shift to all cash may be necessary. If this happens, there is a sense in which the replicating strategy has “failed.” Of course, if the underlying portfolio ends up below where it started, delivering the floor is just what was planned. But if the underlying portfolio ends up above where it started (as shown in the graph), the “insured portfolio” that was stopped-out (the lower jagged line) could end up with a performance much inferior to one that was not stopped-out (the upper jagged line).
It is not enough to replicate an insured portfolio without counting trading costs; trading costs may be so large as to create substantial net losses when included—hardly the motivation behind portfolio insurance. Indeed, often the key difference between portfolio insurance implementations is how they handle trading costs. Both the floor and upside capture should be estimated net of trading costs.

A trick to incorporate trading costs follows from the observation that for portfolio insurance, higher trading costs (proportional to dollar volume) are equivalent to higher volatility. Here is the logic. Following the replicating strategy, one buys in after an increase in the underlying portfolio price. More volatility is like seeing a higher price before buying in, while trading costs are like paying a higher price. On the downside, one sells out. In this case, more volatility is like seeing a lower price before selling out, while trading costs are like receiving a lower price. So, on both the upside and downside, volatility and trading costs are indistinguishable. For a given level of proportional trading costs and frequency of portfolio revision, this logic can be turned into a precise calculation of the higher level of volatility that can be used in a no-trading-costs analysis to correct for the missing costs.

If the basic portfolio insurance strategy is followed scrupulously, it can lead to extremely high trading costs near the payoff date when the underlying portfolio price is trading near the floor (a high gamma region). A way around this is to remove the kink from the payoff line and replace it with a smooth curve in that region. This prevents the slope (delta) from changing too quickly in this region, and only changes the payoff line slightly—a very favorable tradeoff. A trick for doing this is to pretend, counterfactually, that the time to payoff is slightly longer than it actually is.

Trading rules governing how often and how much to revise the replicating portfolio can be quite complex. The more revision, the more accurate the replicating strategy in delivering the portfolio insurance payoff before including trading costs, but the greater these trading costs will be. Somehow, we need to strike an optimal balance between these two conflicting aims: high accuracy and low trading costs. A simple rule is to transact every time the underlying portfolio changes by more than X percent from its last value. Probably a better trading rule is to trade whenever the current delta is different by more than X (in absolute value) from the target delta (the delta we would choose in the absence of trading costs). An even better rule may be to modify this so that X is smaller in high gamma regions.

Not just how often, but how much to trade when we do, is another question. To the extent that trading costs are fixed (not dependent on the amount traded), we will want to trade all the way to the target delta. To the extent that trading costs are proportional, we will want to keep our delta close to but not at the target delta.

The Achilles heel of modern option pricing theory is jumps or gap openings in the price of the underlying asset. The significance of a jump depends largely on when it happens. If it occurs during a low gamma period, it is of little concern, since whether the asset price is where it was before or where it is after the jump has little effect on the desired delta. If the jump occurs when gamma is high, however, serious errors in replication will occur.

For concave payoff lines (where we want to buy in after a fall and sell out after a rise), jumps will increase our profits. We will be holding more than we should (purely for replication purposes) when the asset price rises, and less when it falls. However, for convex payoff lines like portfolio insurance, the opposite is true. Jumps essentially cause the replicating strategy to lag behind unfavorably relative to where it should be. When prices fall (rise), the jump prevents us from selling out (buying in) fast enough.

Jumps are difficult to handle, but here are some ways to control for them. First, modify the payoff line to reduce the severity of high gamma regions. For portfolio insurance, as we have already suggested, replace the kink in the payoff with a smooth curve. Second, supplement dynamic replication with static replication using buy-and-hold options. These are immune to jumps, provided the market mechanism that guarantees their payoff does not fail. Third, to preserve the floor even after a high gamma jump, add “jump protection.” With this modification, the normal replicating strategy is overridden and the insured portfolio is put 100 percent in cash in advance when a jump of a certain preset magnitude could subsequently drive the strategy through the floor.

I learned many of the “tricks” described herein from my long association with my colleague Hayne Leland.

This article was excerpted from Mark Rubinstein’s new book, Derivatives: A PowerPlus Picture Book, available at www.in-the-money.com.