Fitting spectrum and composition of Ultra-High Energy Cosmic Rays

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Fitting spectrum and composition of Ultra-High Energy Cosmic Rays

Overview

• Introduction
• Propagation of Ultra High Energy Cosmic Rays (UHECR)
  • Main factors
  • Interactions
  • Simulations of cosmic rays propagation
• Fitting experimental spectra and composition
• Conclusion
Observed spectrum of cosmic rays

UHECR: $E > 10^{18} \text{eV}$
Observed spectrum of cosmic rays

\[ E^{-3.3} \]

\[ E^{-2.6} \]

HiRes
Auger ICRC 2009

power laws
power laws + smooth function

\[ \sigma_{\text{sys}}(E) = 22\% \]
Fluorescence Detector: Longitudinal Shower Profiles

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Fluorescence Detector: Longitudinal Shower Profiles

\[ E \propto \int (\frac{dE}{dX}) \, dX \]

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Average Shower Maximum $<X_{\text{max}}>$ and \text{RMS}(X_{\text{max}})$

$< X_{\text{max}} > \sim D \log(E/A) + \text{const}$

$\sigma_{X_{\text{max}}}(A_2) < \sigma_{X_{\text{max}}}(A_1)$ for $A_2 > A_1$

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Main Factors influencing UHECR propagation

Radio background (RB)

Microwave Photon Background (MWB)

IR/Optic radiation

Nuclei

Random Extragalactic Magnetic Field (EGMF) $10^{-20} - 10^{-9}$ G

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Interactions

- Protons and neutrons

  Pion production \( N \gamma \rightarrow N' \pi \ldots \)

  \( p \gamma \rightarrow p e^+ e^- \)

  \( n \rightarrow p e^- \bar{\nu}_e \)
Interactions

- **Protons and neutrons**
  
  **Pion production**
  
  \[ N \gamma_b \rightarrow N' \pi \ldots \]
  
  \[ E_{th} = \frac{m_\pi (m_p + m_\pi/2)}{\epsilon} \approx 7 \times 10^{16} \left( \frac{\epsilon}{eV} \right)^{-1} eV \]
  
  For MWB (\( \epsilon \approx 10^{-3} eV \)): \( E_{th} \approx 70 EeV \)

  **e\(^+\)e\(^-\) pair production**
  
  \[ p \gamma_b \rightarrow p e^+ e^- \]

  **Neutron \( \beta \)-decay**
  
  \[ n \rightarrow p e^- \bar{\nu}_e \]
Interactions

- Protons and neutrons

Pion production

\[ N \gamma_b \rightarrow N' \pi \ldots \]

\[ E_{th} = \frac{m_{\pi}(m_p+m_{\pi}/2)}{\epsilon} \approx 7 \times 10^{16} \left( \frac{\epsilon}{eV} \right)^{-1} eV \]  

For MWB (\( \epsilon \approx 10^{-3}eV \)): \( E_{th} \approx 70EeV \)

\[ e^+e^- \text{ pair production} \]

\[ p \gamma_b \rightarrow p e^+ e^- \]

\[ E_{th} = \frac{m_e(m_A+m_e)}{\epsilon} \approx 5 \times 10^{14} \left( \frac{\epsilon}{eV} \right)^{-1} eV \]  

For MWB (\( \epsilon \approx 10^{-3}eV \)): \( E_{th} \approx 5 \times 10^{17}eV \)

Neutron \( \beta \)-decay

\[ n \rightarrow pe^- \bar{\nu}_e \]
Interactions

- **Nuclei**
  - Pion production
  - $A \gamma_b \rightarrow A \pi \ldots$
  - $A \gamma_b \rightarrow A e^+ e^-$
  - Photo-disintegration
  - $A \gamma_b \rightarrow A' \ N_\ldots$
- **Protons and neutrons**
  - Pion production
  - $N \gamma_b \rightarrow N' \pi \ldots$
  - $p \gamma_b \rightarrow p e^+ e^-$
  - e$^+$ e$^-$ pair production
  - Neutron β-decay
  - $n \rightarrow p e^- \bar{\nu}_e$
- **Electron-photon cascade**

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Energy loss lengths

- Proton
- Iron

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Interactions

- Protons, neutrons and nuclei
  - Pion production: $A \gamma_b \rightarrow A \pi \ldots$
  - $e^+ e^-$ pair production: $A \gamma_b \rightarrow A e^+ e^-$
  - Photo-disintegration: $A \gamma_b \rightarrow A' N$
  - Neutron $\beta$-decay: $n \rightarrow p e^- \nu_e$

- Electron-photon cascade
  - Inverse Compton: $e \gamma_b \rightarrow e \gamma$
  - $e^+ e^-$ pair production: $\gamma \gamma_b \rightarrow e^+ e^-$

$$E_{th} = \frac{m_e^2}{\epsilon} \simeq 2.6 \times 10^{11} \left( \frac{\epsilon}{eV} \right)^{-1} eV$$

For MWB ($\epsilon \simeq 10^{-3} eV$): $E_{th} \simeq 5 \times 10^{14} eV$

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Interactions

- **Protons, neutrons and nuclei**
  - Pion production: $A \gamma_b \rightarrow A \pi \ldots$
  - $e^+ e^-$ pair production: $A \gamma_b \rightarrow A e^+ e^-$
  - Photo-disintegration: $A \gamma_b \rightarrow A' N..$
  - Neutron $\beta$-decay: $n \rightarrow pe^- \bar{\nu}_e$

- **Electron-photon cascade**
  - Inverse Compton: $\epsilon \gamma_b \rightarrow \epsilon \gamma$
  - $e^+ e^-$ pair production: $\gamma \gamma_b \rightarrow e^+ e^-$
  - Synchrotron losses
  - Double pair production: $\gamma \gamma_b \rightarrow e^+ e^- e^+ e^-$
  - $e^+ e^-$ pair production by $e$: $e \gamma_b \rightarrow e^+ e^- e^+ e^-$

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Interactions

- Electron-photon cascade
  
  **Inverse Compton**
  \[ e \gamma \rightarrow e \gamma \]
  \[ \gamma \gamma \rightarrow e^+ e^- \]

  **e^+ e^- pair production**
  \[ e \gamma \rightarrow e^+ e^- \]

  **Synchrotron losses**
  \[ \gamma \gamma \rightarrow e^+ e^- \]

  **Double pair production**
  \[ \gamma \gamma \rightarrow e^+ e^- e^+ e^- \]

  **e^+ e^- pair production by e**
  \[ e \gamma \rightarrow e^+ e^- \]

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Deflection and synchrotron radiation

Gyroradius: \[ R_g = \frac{E}{q e B} \approx 110 \times \frac{1}{Z} \left( \frac{E}{10^{19} \text{eV}} \right) \left( \frac{10^{-10} \text{G}}{B} \right)^{-1} \text{Mpc} \]

Synchrotron loss length:

\[ \frac{dE}{dt} = -\frac{4}{3} \sigma_T \frac{B^2}{8\pi} \left( \frac{q m_e}{m} \right)^4 \left( \frac{E}{m_e} \right)^2 \]

\[ E_\gamma \approx \frac{3eB}{2m_e} \left( \frac{E_e}{m_e} \right)^2 \approx \]

\[ 2.2 \times 10^{14} \left( \frac{E_e}{10^{21} \text{eV}} \right)^2 \left( \frac{B}{10^{-9} \text{G}} \right) \text{eV} \]

The gyroradius and the synchrotron loss rates of electrons for various strengths of the EGMF

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
### Some references on UHECR propagation

<table>
<thead>
<tr>
<th>Topic</th>
<th>Reference</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Astropart. Phys. 23 (2005) 191-201</td>
</tr>
<tr>
<td>Extragalactic magnetic field</td>
<td>K. Dolag et al., astro-ph/0410419</td>
</tr>
<tr>
<td>Infrared background</td>
<td>F. Stecker et al. astro-ph/0510449</td>
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<tr>
<td>Radio background</td>
<td>T.A. Clark, L.W. Brown, and J.K. Alexander, Nature 228, 847</td>
</tr>
<tr>
<td></td>
<td>R.J. Protheroe, P.L. Biermann, Astropart. Phys. 6, 45</td>
</tr>
</tbody>
</table>

**Fitting spectrum and composition of Ultra-High Energy Cosmic Rays**
Simulations of cosmic rays propagation

- Monte Carlo based simulations
  - Random extragalactic magnetic field is taken into account

- Transport equation approach (rectilinear propagation)
  - Fast calculation (good for parameter space scanning)
  - Gives correct result for
    \[ E \geq 10^{17} \text{eV} \times Z \times \frac{B}{10^{-10} \text{G}}, \quad L_{\text{cor}} = 1 \text{Mpc} \]
    or for homogeneous source distribution if distance to the closest source is less than diffuse length

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Simulations of cosmic rays propagation

Sample transport equation for electrons (includes only pair production PP and inverse Compton scattering ICS)

\[
\frac{d}{dt} N_e(E_e, t) = -N_e(E_e, t) \int d\epsilon n(\epsilon) \int d\mu \frac{1 - \beta'_{\epsilon \mu}}{2} \sigma_{ICS}(E_e, \epsilon, \mu) + \\
\int dE'_{e} N_e(E'_e, t) \int d\epsilon n(\epsilon) \int d\mu \frac{1 - \beta'_{\epsilon \mu}}{2} \frac{d\sigma_{ICS}}{dE_e}(E_e; E'_e, \epsilon, \mu) + \\
\int dE_\gamma N_\gamma(E_\gamma, t) \int d\epsilon n(\epsilon) \int d\mu \frac{1 - \mu}{2} \frac{d\sigma_{PP}}{dE_e}(E_e; E_\gamma, \epsilon, \mu) + Q(E_e, t)
\]
Fitting experimental data

- Energy spectrum $j(E)$
- Chemical composition
  - Average Shower Maximum $\langle X_{\text{max}} \rangle (E)$
  - Shower-to-Shower Fluctuations $\sigma(X_{\text{max}}) (E)$
Fitting experimental data

- Energy spectrum $j(E)$
  - Binned maximum likelihood function is used
  - Poisson probability of the observed event set is maximized
    \[
    L(n; \nu) = \prod_{i}^{N} \frac{n_{i}^{\nu_{i}}}{\nu_{i}!} e^{\nu_{i}}
    \]
  - Goodness of fit defined as fraction of hypothetical experiments which result in worse agreement with the theory than the real data having the same total number of events

Phenomenological source model:

\[
F(E, z) = f E^{-\alpha} \text{Exp}(-E/E_{\text{max}}) (1+z)^{3+m} \Theta(z-z_{\text{min}}) \Theta(z_{\text{max}} - z)
\]

$z$ – red shift, $\Theta(x)$-step function
Phenomenological source model:

\[ F(E, z) = f E^{-\alpha} \exp\left(-\frac{E}{E_{\text{max}}}\right) (1+z)^{3+m} \Theta(z-z_{\text{min}}) \Theta(z_{\text{max}}-z) \]

\( z \) – red shift, \( \Theta(x) \)-step function

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Typical Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Power of the Injection Spectrum, ( E^{-\alpha} )</td>
<td>( \alpha )</td>
<td>( 1 \leq \alpha \leq 2.7 )</td>
</tr>
<tr>
<td>End point of the Energy Spectrum</td>
<td>( E_{\text{max}} )</td>
<td>( 2 \times 10^{20} \leq E_{\text{max}} \leq 10^{21} )</td>
</tr>
<tr>
<td>Evolution factor: ( (1+z)^{3+m} )</td>
<td>( m )</td>
<td>( 0 \leq m \leq 4 )</td>
</tr>
<tr>
<td>Red shift of the nearest source</td>
<td>( z_{\text{min}} )</td>
<td>( 0 &lt; Z_{\text{min}} &lt; 0.01 )</td>
</tr>
<tr>
<td>Maximal source redshift</td>
<td>( z_{\text{max}} )</td>
<td>( 3 &lt; Z_{\text{max}} &lt; 6 )</td>
</tr>
</tbody>
</table>

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Fitting experimental data

**Chemical composition**
- $<X_{\text{max}}> (E)$
- $\sigma(X_{\text{max}}) (E)$

For mixed composition:

$$< X_{\text{max}} > \approx D \log(E/A) + \text{const}$$

For mixed composition:

$$< X_{\text{max}} > = \sum A \frac{N_A}{N_{\text{tot}}} < X_{\text{max}} >_A$$

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Fitting experimental data

- Chemical composition
  - $<X_{\text{max}}> (E)$
  - $\sigma(X_{\text{max}}) (E)$

$$\sigma_A(X_{\text{max}}) \simeq \sigma_p(X_{\text{max}}) A^{-\alpha}$$
$$\alpha \simeq 0.2 \text{ for QGSJET01}$$

For mixed composition:

$$RMS(X_{\text{max}})^2 = \sum_A \frac{N_A}{N_{\text{tot}}} RMS_A(X_{\text{max}})^2$$

$$\sigma(X_{\text{max}})^2 = RMS(X_{\text{max}})^2 - <X_{\text{max}}>^2$$

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Joint fit of the spectrum and composition

$\chi^2$ statistics is used to obtain goodness of joint fit

Bins with small number of events are combined into larger bins

Goodness of spectrum fit in the bins with small number of events is calculated separately

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Spectrum and composition fitting examples

**Fe + p**

\[ E_{\text{fit}} \geq 8EeV \]

p:51%  Fe:49%

\( \alpha = 2; E_{\text{max}} = Z25EeV; m = -2 \)

Spectrum & Xmax fit goodness 0.31

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Spectrum and composition fitting examples

\[ E_{fit} \geq 8E_{eV} \]

p:51%   Fe:49%

\[ \alpha = 2; E_{max} = Z^{25}E_{eV}; m = -2 \]

Spectrum & Xmax fit goodness 0.31

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Spectrum and composition fitting examples

Mixed Composition Model (using galactic abundances)

\[ p: 13\% \text{ mixed:} 87\% \quad E_{fit} \geq 8 E_{eV} \]
\[ \alpha = 0.5; E_{max} = Z 4 E_{eV}; m = -2 \]

Joint fit goodness 0.38
Energy scale shifted by factor 1.37

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Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Spectrum and composition fitting examples

Mixed Composition Model
(using galactic abundances)

\[ p: 13\% \quad \text{mixed:} 87\% \quad E_{fit} \geq 8EeV \]
\[ \alpha = 0.5; E_{max} = Z4EeV; m = -2 \]

Energy scale shifted by factor 1.37

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Spectrum and composition fitting examples

mixed + p + He \quad E_{fit} \geq 8EeV

p: 4%  He: 11%  mixed:85%

\alpha = 0.5; \quad E_{max} = Z4EeV; \quad m = -2

Energy scale shifted by factor 1.37

Fitting spectrum and composition of Ultra-High Energy Cosmic Rays
Fitting spectrum and composition of Ultra-High Energy Cosmic Rays

Dan Hooper et al. astro-ph: 0910.1842v1

90% N and 10% Fe

70% N and 30% Si  \( B = 0.3nG, L_{\text{cor}} = 1Mpc \)
Conclusions

- Auger spectrum can not be fitted with homogeneously distributed pure proton source
- Fitting <Xmax> & RMS(Xmax) along with spectrum shape strongly constrain possible models of cosmic ray sources
- UHECR source distribution effects may be important
FIG. 1: Observational limits on EGMF. Cyan shaded region shows the upper limit on $B$ imposed by the Zeeman splitting measurement, the lower bound on the correlation length imposed by the magnetic diffusion and the upper bound on correlation length given by the Hubble radius. Orange shaded region shows the limit from Faraday rotation measurements. Filled orange region shows the limit derived in the Ref. [23], while the orange-hatched region is the limit derived in the Ref. [21]. Magenta line shows limit which can be imposed by observations of deflections of UHECR [24]. Violet vertical-hatched regions and the arrows at $\lambda_B \sim 0.5$ Mpc and $\lambda_B \sim R_H$ show the limits imposed on cosmologically produced fields by the CMB observations [36, 37, 40, 45]. Black ellipses show the ranges of measured magnetic fields in galaxies and galaxy clusters.

FIG. 2: Model predictions and estimates for the EGMF strength. Cyan shaded region and black ellipses show the experimental limits and measurements from Fig. 2. Upper bound at $B \sim 10^{-10}$ G shown by solid line comes from flux conservation during galaxy formation argument \[1\]. Upper bound at $B = 10^{-12}$ G shows a limit imposed by constrained simulations of magnetic fields in galaxy clusters \[12, 33\]. Left panel: left and right hatched regions show theoretically allowed range of values of $B, \lambda_B$ for non-helical and helical fields generated at the epoch of electroweak phase transition during radiation-dominated era. Middle panel: left and right hatched region show ranges of possible $B, \lambda_B$ for nonhelical and helical magnetic fields produced during the QCD phase transition. Right panel: hatched region is the range of possible $B, \lambda_B$ for EGMF generated during recombination epoch. Dark grey shaded region shows the range of $(B, \lambda_B)$ parameter space accessible for the $\gamma$-ray measurements via $\gamma$-ray observations. Light-grey shaded regions show the parts of the parameter space in which the existence of EGMF could be confirmed or ruled out, but no measurements of EGMF strength is possible.