Interplay of flavour and CP symmetries

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Outline

• lepton mixing: parametrization and data
• combination of flavour and CP symmetries
  • general idea
  • examples: $G_f = S_4$ and $G_f = \Delta(48)$
  • predictions for leptogenesis
• conclusions & outlook
Parametrization of lepton mixing

• charged lepton and (Majorana) neutrino mass terms

\[ e^c_a m_{e,ab} l_b \quad \text{and} \quad \nu_a m_{\nu,ab} \nu_b \]

cannot be diagonalized simultaneously

• going to the mass basis

\[ U^\dagger_e m_e U_e = \text{diag}(m_e^2, m_\mu^2, m_\tau^2) \quad \text{and} \quad U^T_\nu m_\nu U_\nu = \text{diag}(m_1, m_2, m_3) \]

leads to non-diagonal charged current interactions

\[ \bar{l} \gamma^\mu W^{-} U_{PMNS} \nu \quad \text{with} \quad U_{PMNS} = U_e^\dagger U_\nu \]
Parametrization of lepton mixing

Parametrization \textbf{(PDG)} \[ U_{PMNS} = \tilde{U} \, \text{diag}(1, e^{i\alpha/2}, e^{i(\beta/2+\delta)}) \]

with

\[
\tilde{U} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
    -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\
    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix}
\]

and \( s_{ij} = \sin \theta_{ij}, \ c_{ij} = \cos \theta_{ij} \)

Jarlskog invariant \( J_{CP} \)

\[
J_{CP} = \text{Im} \left[ U_{PMNS,11}U_{PMNS,13}^*U_{PMNS,31}^*U_{PMNS,33} \right] \\
= \frac{1}{8} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13} \cos \theta_{13} \sin \delta
\]
Parametrization of lepton mixing

Parametrization \((\text{PDG})\)

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\end{pmatrix}
\]

and \(s_{ij} = \sin \theta_{ij}, \ c_{ij} = \cos \theta_{ij}\)

Majorana invariant \(I_1\)

\[
I_1 = \text{Im} \left[ U_{PMNS,12}^2 \left( U_{PMNS,11}^* \right)^2 \right]
\]

\[
= \sin^2 \theta_{12} \cos^2 \theta_{12} \cos^4 \theta_{13} \sin \alpha
\]
Parametrization of lepton mixing

Parametrization \textbf{(PDG)}

\[ U_{PMNS} = \tilde{U} \text{ diag}(1, e^{i\alpha/2}, e^{i(\beta/2 + \delta)}) \]

with

\[ \tilde{U} = \begin{pmatrix}
    c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\
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    s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13}
\end{pmatrix} \]

and \( s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij} \)

Majorana invariant \( I_2 \)

\[ I_2 = \text{Im} \left[ U_{PMNS,13}^2 (U_{PMNS,11}^*)^2 \right] = \sin^2 \theta_{13} \cos^2 \theta_{12} \cos^2 \theta_{13} \sin \beta \]
Data on lepton mixing

Latest global fits

(Capozzi et al. (’13))
Data on lepton mixing

Latest global fits

(Capozzi et al. ('13))
Data on lepton mixing

Latest global fits NH [IH] (Capozzi et al. ('13))

**best fit and 1 σ error**

\[
\begin{align*}
\sin^2 \theta_{13} &= 0.0234^{+0.0022}_{-0.0018} \\
\sin^2 \theta_{12} &= 0.308^{+0.017}_{-0.017} \\
\sin^2 \theta_{23} &= \begin{cases} 
0.425^{+0.029}_{-0.027} \\
[0.531 \leq \sin^2 \theta_{23} \leq 0.610]
\end{cases} \\
\delta &= 1.39^{+0.33}_{-0.27}\pi
\end{align*}
\]

**3 σ range**

\[
\begin{align*}
0.0177^{+0.0022}_{-0.0018} \leq \sin^2 \theta_{13} \leq 0.0297^{+0.017}_{-0.017} \\
0.259 \leq \sin^2 \theta_{12} \leq 0.359 \\
0.357^{+0.029}_{-0.027} \leq \sin^2 \theta_{23} \leq 0.641^{+0.027}_{-0.027} \\
0 \leq \delta \leq 2\pi
\end{align*}
\]

\(\alpha, \beta\) unconstrained
Data on lepton mixing

Latest global fits \( \text{NH [IH]} \) \((\text{Capozzi et al. ('13)})\)

\[
\left| U_{PMNS} \right| \approx \begin{pmatrix}
0.82 & 0.55 & 0.15 \\
0.40[39] & 0.65 & 0.64[5] \\
0.40[2] & 0.52 & 0.75[4]
\end{pmatrix}
\]

and no information on Majorana phases

\[
\downarrow
\]

Mismatch in lepton flavour space is large!
General idea

• interpret this mismatch in lepton flavour space as mismatch of residual symmetries $G_\nu$ and $G_e$

• if we want to predict lepton mixing, we have to derive this mismatch

• let us assume that there is a symmetry, broken to $G_\nu$ and $G_e$

• this symmetry is in the following a combination of a finite, discrete, non-abelian symmetry $G_f$ and CP

(Proglio et al. (’12,’13), Holthausen et al. (’12), Grimus/Rebelo (’95))

[Masses do not play a role in this approach.]
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[Masses do not play a role in this approach.]
General idea

Idea:
Relate lepton mixing to how $G_f$ and $CP$ are broken
Interpretation as mismatch of embedding of different sub-groups $G_\nu$ and $G_e$ into $G_f$ and $CP$

\[
\begin{array}{c}
G_f & \text{& CP} \\
\downarrow & \downarrow \\
G_\nu & \quad G_e \\
\text{neutrinos} & \text{charged leptons}
\end{array}
\]
General idea

Idea:
Relate lepton mixing to how $G_f$ and $CP$ are broken
Interpretation as mismatch of embedding of different sub-groups $G_\nu$ and $G_e$ into $G_f$ and $CP$

$G_f$ & $CP$

\[ \begin{align*}
\uparrow & \quad \uparrow \\
\text{neutrinos} & \quad \text{charged leptons} \\
\text{assume 3 generations} & \quad \text{distinguish 3 generations}
\end{align*} \]

of Majorana neutrinos
General idea

Idea:
Relate lepton mixing to how $G_f$ and $CP$ are broken
Interpretation as mismatch of embedding of different sub-groups $G_\nu$ and $G_e$ into $G_f$ and $CP$

\[ G_f \text{ & } CP \]

\[
\begin{align*}
\downarrow & \quad \downarrow \\
\text{neutrinos} & \quad \text{charged leptons} \\
G_\nu & = Z_2 \times CP \\
G_e & = Z_N \quad \text{with} \quad N \geq 3
\end{align*}
\]

An example: $\mu \tau$ reflection symmetry \((Harrison/Scott (’02,’04), Grimus/Lavoura (’03))\)
**General idea**

\[ G_f & \text{CP} \]

\[ \downarrow \quad \downarrow \]

neutrinos \[ G_\nu = Z_2 \times \text{CP} \]

charged leptons \[ G_e = Z_N \quad \text{with} \quad N \geq 3 \]

**Further requirements**

- two/three non-trivial mixing angles \( \Rightarrow \) irred 3-dim rep of \( G_f \)
- "maximize" predictability of approach
General idea

Definition of generalized CP transformation \( \text{(see e.g. Branco et al. ('11))} \)

\[ \phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^* \]

with \( X \) is unitary and symmetric
**General idea**

Definition of generalized CP transformation *(see e.g. Branco et al. ('11))*

\[ \phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^* \]

with \( X \) is unitary and symmetric;

apply \( CP \) twice

\[ \phi \xrightarrow{\text{CP}} X \phi^* \xrightarrow{\text{CP}} XX^* \phi = \phi \]
General idea

Definition of generalized CP transformation \( \text{see e.g. Branco et al. ('11)} \)

\[
\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*
\]

with \( X \) is unitary and symmetric.

Realize direct product of \( Z_2 \subset G_f \) and \( CP \)
General idea

Definition of generalized CP transformation (see e.g. Branco et al. (’11))

\[ \phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^* \]

with \( X \) is unitary and symmetric.
Realize direct product of \( Z_2 \subset G_f \) and \( CP \); \( Z \) generates \( Z_2 \)

\[ \phi \xrightarrow{\text{CP}} X \phi^* \xrightarrow{Z_2} XZ^* \phi^* \quad \text{and} \quad \phi \xrightarrow{Z_2} Z \phi \xrightarrow{\text{CP}} ZX \phi^* \]
General idea

Definition of generalized CP transformation (see e.g. Branco et al. ('11))

\[ \phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^* \]

with \( X \) is unitary and symmetric.
Realize direct product of \( Z_2 \subset G_f \) and CP; \( Z \) generates \( Z_2 \)

\[ X Z^* - ZX = 0 \]
General idea

• neutrino sector: $Z_2 \times \text{CP preserved}$
  - neutrino mass term $\nu_a m_{\nu,ab} \nu_b$
    - is invariant under $\nu_\alpha \rightarrow Z_{\alpha\beta} \nu_\beta$
  - is invariant under generalized CP transformation $\nu_\alpha \rightarrow X_{\alpha\beta} \nu_\beta^*$

• charged lepton sector: $Z_N, N \geq 3$, preserved
  - charged lepton mass term $e_c^a m_{e,ab} l_b$
    - is invariant under $l_\alpha \rightarrow Q_{e,\alpha\beta} l_\beta$
General idea

• neutrino sector: $Z_2 \times \text{CP} \text{ preserved}$
  \[ Z^T m_\nu Z = m_\nu \quad \text{and} \quad X m_\nu X = m^*_\nu \]

• charged lepton sector: $Z_N, N \geq 3, \text{ preserved}$
  \[ Q^\dagger e^\dagger m_e Q_e = m^\dagger e m_e \]
General idea

• neutrino sector: $\mathbb{Z}_2 \times \text{CP} \text{ preserved and generated by } (\nu = \Omega_{\nu}, \nu')$

\[
X = \Omega_{\nu}\Omega_{\nu}^T \quad \text{and} \quad Z = \Omega_{\nu}Z^{\text{diag}}\Omega_{\nu}^\dagger
\]

$Z^{\text{diag}} = \text{diag}(-1, 1, -1)$ and $\Omega_{\nu}$ unitary

• charged lepton sector: $Z_N$, $N \geq 3$, preserved

$\rightarrow$ charged lepton mass matrix $m_e$ fulfills

\[
Q_e^\dagger m_{e}^\dagger m_e Q_e = m_{e}^\dagger m_e
\]
General idea

• neutrino sector: $Z_2 \times \text{CP preserved}$
  \[ \rightarrow \text{neutrino mass matrix } m_\nu \text{ fulfills} \]
  \[ Z^{\text{diag}}[\Omega^T_{\nu} m_\nu \Omega_{\nu}] Z^{\text{diag}} = [\Omega^T_{\nu} m_\nu \Omega_{\nu}] \quad \text{and} \quad [\Omega^T_{\nu} m_\nu \Omega_{\nu}] = [\Omega^T_{\nu} m_\nu \Omega_{\nu}]^* \]

• charged lepton sector: $Z_N$, $N \geq 3$, preserved
  \[ \rightarrow \text{charged lepton mass matrix } m_e \text{ fulfills} \]
  \[ Q^\dagger_e m^\dagger_e m_e Q_e = m^\dagger_e m_e \]
**General idea**

- neutrino sector: $Z_2 \times \text{CP} \text{ preserved}$
  
  $\rightarrow$ neutrino mass matrix $m_\nu$ is diagonalized by
  
  $$\Omega_\nu(X, Z) R(\theta) K_\nu$$

- charged lepton sector: $Z_N, N \geq 3, \text{ preserved}$
  
  $\rightarrow$ charged lepton mass matrix $m_e$ fulfills
  
  $$Q_e^\dagger m_e^\dagger m_e Q_e = m_e^\dagger m_e$$
General idea

• neutrino sector: $Z_2 \times \text{CP preserved}$
  $\rightarrow$ neutrino mass matrix $m_\nu$ is diagonalized by
  \[ \Omega_\nu(X, Z) R(\theta) K_\nu \]

• charged lepton sector: $Z_N$, $N \geq 3$, preserved and generated by
  \[ Q_e = \Omega_e Q_e^{diag} \Omega_e^\dagger \text{ with } \Omega_e \text{ unitary} \]
  \[ Q_e^{diag} = \text{diag} (\omega^e_N, \omega^\mu_N, \omega^\tau_N) \]
  and $n_e \neq n_\mu \neq n_\tau$ and $\omega_N = e^{2\pi i/N}$
General idea

• neutrino sector: $Z_2 \times \text{CP}$ preserved
  $\rightarrow$ neutrino mass matrix $m_\nu$ is diagonalized by
  \[ \Omega_\nu(X, Z) R(\theta) K_\nu \]

• charged lepton sector: $Z_N$, $N \geq 3$, preserved
  $\rightarrow$ charged lepton mass matrix $m_e$ fulfills
  \[ \Omega_e^\dagger(Q_e)m_e^\dagger m_e \Omega_e(Q_e) \text{ is diagonal} \]
General idea

- neutrino sector: $\mathbb{Z}_2 \times \text{CP}$ preserved
  - neutrino mass matrix $m_\nu$ is diagonalized by
    \[
    \Omega_\nu(X, Z) R(\theta) K_\nu
    \]
- charged lepton sector: $\mathbb{Z}_N$, $N \geq 3$, preserved
  - charged lepton mass matrix $m_e$ fulfills
    \[
    \Omega^\dagger_e(Q_e) m_e^\dagger m_e \Omega_e(Q_e) \text{ is diagonal}
    \]
- conclusion: PMNS mixing matrix reads
  \[
  U_{PMNS} = \Omega^\dagger_e \Omega_\nu R(\theta) K_\nu \text{ in } \bar{l} W - U_{PMNS} \nu
  \]
General idea

\[ U_{PMNS} = \Omega_e^\dagger \Omega_\nu R(\theta) K_\nu \]

- 3 unphysical phases are removed by \( \Omega_e \rightarrow \Omega_e K_e \)
- \( U_{PMNS} \) contains one parameter \( \theta \)
- permutations of rows and columns of \( U_{PMNS} \) possible

\[ \downarrow \]

**Predictions:**
Mixing angles and CP phases are predicted in terms of one parameter \( \theta \) only, up to permutations of rows/columns
General idea: consistency conditions

We want to consistently combine $G_f$ and the generalized CP transformation $\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*$

$\Downarrow$

"closure" relations have to hold
General idea: consistency conditions

We want to consistently combine $G_f$ and the generalized CP transformation

$\phi_i \xrightarrow{\text{CP}} X_{ij} \phi^*_j$

\[ \downarrow \]

"closure" relations have to hold:

assume $\phi$ transforms as 3-dim rep of $G_f$, then

$\phi \xrightarrow{\text{CP}} X\phi^* \xrightarrow{G_f} XA^*\phi^* \xrightarrow{\text{CP}} XA^*X^*\phi = (X^*AX)^* \phi$
General idea: consistency conditions

We want to consistently combine $G_f$ and the generalized CP transformation $\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*$

"closure" relations have to hold:

$$(X^*AX)^* = A'$$ with in general $A \neq A'$ and $A, A' \in G_f$$
General idea: consistency conditions

We want to consistently combine $G_f$ and the generalized CP transformation $\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*$

\[ \Downarrow \]

"closure" relations have to hold:

\[(X^*AX)^* = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A, A' \in G_f\]

compare to relation for having direct product of $Z_2$ and $CP$

\[XZ^* - ZX = 0\]
General idea: consistency conditions

We want to consistently combine $G_f$ and the generalized CP transformation $\phi_i \xrightarrow{\text{CP}} X_{ij} \phi_j^*$

\[ \Downarrow \]

"closure" relations have to hold:

\[ (X^*AX)^* = A' \quad \text{with in general} \quad A \neq A' \quad \text{and} \quad A, A' \in G_f \]

compare to relation for having direct product of $Z_2$ and CP

\[ (X^*ZX)^* = Z \]
General idea: consistency conditions

- fulfilling these conditions ensures a consistent theory, but can lead to enlarged symmetry group
- additional requirement in order not to change representation content of \( G_f \) \((Chen \ et \ al. \ ('14))\):
  - all representations transform into complex conjugate under \( CP \)
General idea: consistency conditions

• fulfilling these conditions ensures a consistent theory, but can lead to enlarged symmetry group
• additional requirement in order not to change representation content of $G_f$ \((Chen \ et \ al. \ (’14))\):

  all representations transform into complex conjugate under $CP$

  [mathematically: mapping induced via $X$ has to be ’class-inverting’ automorphism ($A' \sim A^{-1}$)]
General idea: consistency conditions

- fulfilling these conditions ensures a consistent theory, but can lead to enlarged symmetry group
- additional requirement in order not to change representation content of $G_f$ (Chen et al. ('14)):
  
  all representations transform into complex conjugate under $CP$

- if not fulfilled or not possible to fulfill for $G_f$
  $\Rightarrow$ constraints on representations
  $[S_4$ fulfilled;
  $\Delta(48)$ not fulfilled in general, only for certain representations]
Study of $S_4$ and $CP$

Generators in rep. $3'$:

$\omega = e^{2\pi i/3}$

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega^2 & 0 \\ 0 & 0 & \omega \end{pmatrix}, \quad U = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfill

$S^2 = 1$, $T^3 = 1$, $U^2 = 1$, $(ST)^3 = 1$, $(SU)^2 = 1$, $(TU)^2 = 1$, $(STU)^4 = 1$
Study of $S_4$ and $CP$

A transformation $X$ in rep. $3'$ for $Z = S$ is

$$X_{3'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$XX^\dagger = XX^* = 1$$

$$(X^*AX)^* = A' \ , \ XZ^* - ZX = 0$$
Study of $S_4$ and $CP$

A transformation $X$ in rep. $3'$ for $Z = S$ is

$$X_{3'} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$XX^\dagger = XX^* = 1$$

$$(X^*AX)^* = A'$$

$$XZ^* - ZX = 0$$

Residual symmetry $G_e$ is generated by $T$. 
Study of $S_4$ and $CP$

Maximal $\theta_{23}$ and $\delta$ from $G_e = Z_3$, $Z = S$ and $X_3$,

(Harrison/Scott (02,04), Grimus/Lavoura (03), Feruglio et al. (12,13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix}
2 \cos \theta & \sqrt{2} & 2 \sin \theta \\
- \cos \theta + i \sqrt{3} \sin \theta & \sqrt{2} & - \sin \theta - i \sqrt{3} \cos \theta \\
- \cos \theta - i \sqrt{3} \sin \theta & \sqrt{2} & - \sin \theta + i \sqrt{3} \cos \theta
\end{pmatrix} K_\nu$$

$$\sin^2 \theta_{13} = \frac{2}{3} \sin^2 \theta, \quad \sin^2 \theta_{12} = \frac{1}{2 + \cos 2\theta}, \quad \sin^2 \theta_{23} = \frac{1}{2}$$

and

$$|\sin \delta| = 1, \quad |J_{CP}| = \frac{|\sin 2\theta|}{6 \sqrt{3}}, \quad \sin \alpha = 0, \quad \sin \beta = 0$$
Study of $S_4$ and $CP$

Maximal $\theta_{23}$ and $\delta$ from $G_e = Z_3$, $Z = S$ and $X_3'$

(Harrison/Scott ('02,'04), Grimus/Lavoura ('03), Feruglio et al. ('12,'13))

$$U_{PMNS} = \frac{1}{\sqrt{6}} \begin{pmatrix} 2 \cos \theta & \sqrt{2} & 2 \sin \theta \\ - \cos \theta + i \sqrt{3} \sin \theta & \sqrt{2} & - \sin \theta - i \sqrt{3} \cos \theta \\ - \cos \theta - i \sqrt{3} \sin \theta & \sqrt{2} & - \sin \theta + i \sqrt{3} \cos \theta \end{pmatrix} K_\nu$$

$$\sin^2 \theta_{13} \approx 0.023 \ , \ \sin^2 \theta_{12} \approx 0.341 \ , \ \sin^2 \theta_{23} = \frac{1}{2}$$

and

$$| \sin \delta | = 1 \ , \ |J_{CP}| \approx 0.0348 \ , \ \sin \alpha = 0 \ , \ \sin \beta = 0 \ \text{for} \ \theta \approx 0.185$$
Study of $S_4$ and $CP$

Maximal $\theta_{23}$ and $\delta$ from $G_e = Z_3, Z = S$ and $X_3'$. (Feruglio et al. ('12,'13))
Study of $S_4$ and $CP$

Maximal $\theta_{23}$ and $\delta$ from $G_e = Z_3$, $Z = S$ and $X_3'$ (Feruglio et al. ('12, '13))
Study of $S_4$ and $CP$

Maximal $\theta_{23}$ and $\delta$ from $G_e = Z_3$, $Z = S$ and $X_3'$ (Feruglio et al. ('12,'13))
Study of $\Delta(48)$ and $CP$

Generators in rep. 3:

$\omega = e^{2\pi i / 3}$

(Miller et al. ('16), Fairbairn et al. ('64), Luhn et al. ('07))

\[
a = \begin{pmatrix}
1 & 0 & 0 \\
0 & \omega & 0 \\
0 & 0 & \omega^2
\end{pmatrix}, \quad c = \frac{1}{3} \begin{pmatrix}
1 & 1 - \sqrt{3} & 1 + \sqrt{3} \\
1 + \sqrt{3} & 1 & 1 - \sqrt{3} \\
1 - \sqrt{3} & 1 + \sqrt{3} & 1
\end{pmatrix}, \quad d = a^{-1}ca
\]

which satisfy

\[
a^3 = 1, \quad c^4 = 1, \quad d^4 = 1, \\
\quad cd = dc, \quad aca^{-1} = c^{-1}d^{-1}
\]
Study of $\Delta(48)$ and $CP$

A transformation $X$ in rep. 3 for $Z = c^2$ is \hfill (Ding/Zhou ('13))

$$X_3 = d \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

which fulfills

$$XX^\dagger = XX^* = 1$$
$$\left(X^*AX\right)^* = A' \quad , \quad XZ^* - ZX = 0$$

Residual symmetry $G_e$ is generated by $a$. 
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$  \textit{(Ding/Zhou (’13))}

\[
||U_{PMNS}|| = \frac{1}{\sqrt{6}} \begin{pmatrix}
\frac{1}{\sqrt{2}} \sqrt{4 - (\sqrt{2} + \sqrt{6}) \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{4 + (\sqrt{2} + \sqrt{6}) \cos 2\theta} \\
\frac{1}{\sqrt{2}} \sqrt{4 + (-\sqrt{2} + \sqrt{6}) \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{4 - (-\sqrt{2} + \sqrt{6}) \cos 2\theta} \\
\sqrt{2 + \sqrt{2} \cos 2\theta} & \sqrt{2} & \sqrt{2 - \sqrt{2} \cos 2\theta}
\end{pmatrix}
\]

\[
\sin^2 \theta_{13} = \frac{1}{12} \left(4 + (\sqrt{2} + \sqrt{6}) \cos 2\theta\right), \quad \sin^2 \theta_{12} = \frac{4}{8 - (\sqrt{2} + \sqrt{6}) \cos 2\theta},
\]

\[
\sin^2 \theta_{23} = \frac{1}{2} \left(1 + \frac{\sqrt{6}(\sqrt{3} - 1) \cos 2\theta}{8 - (\sqrt{2} + \sqrt{6}) \cos 2\theta}\right), \quad |J_{CP}| = \frac{\sin 2\theta}{6\sqrt{3}}
\]
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$ (Ding/Zhou ('13))

$$
||U_{PMNS}|| = \frac{1}{\sqrt{6}} \begin{pmatrix}
\frac{1}{\sqrt{2}} \sqrt{4 - (\sqrt{2} + \sqrt{6}) \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{4 + (\sqrt{2} + \sqrt{6}) \cos 2\theta} \\
\frac{1}{\sqrt{2}} \sqrt{4 + (-\sqrt{2} + \sqrt{6}) \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{4 - (-\sqrt{2} + \sqrt{6}) \cos 2\theta} \\
\frac{1}{\sqrt{2}} \sqrt{2 + \sqrt{2} \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{2 - \sqrt{2} \cos 2\theta}
\end{pmatrix}
$$

$$
|\sin \alpha| = \left| \frac{1 + \sqrt{3} - 2\sqrt{2} \cos 2\theta + (-1 + \sqrt{3}) \sin 2\theta}{-4 + (\sqrt{2} + \sqrt{6}) \cos 2\theta} \right|,
$$

$$
|\sin \beta| = \left| \frac{2 \sin 2\theta}{-4 + (2 + \sqrt{3}) \cos^2 2\theta} \right|
$$
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$ \textit{(Ding/Zhou ('13))}

$$\|U_{PMNS}\| = \frac{1}{\sqrt{6}} \begin{pmatrix} \frac{1}{\sqrt{2}} \sqrt{4 - (\sqrt{2} + \sqrt{6}) \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{4 + (\sqrt{2} + \sqrt{6}) \cos 2\theta} \\ \frac{1}{\sqrt{2}} \sqrt{4 + (-\sqrt{2} + \sqrt{6}) \cos 2\theta} & \sqrt{2} & \frac{1}{\sqrt{2}} \sqrt{4 - (-\sqrt{2} + \sqrt{6}) \cos 2\theta} \end{pmatrix}$$

$$\sin^2 \theta_{13} \approx 0.023 \ , \ \sin^2 \theta_{12} \approx 0.341 \ , \ \sin^2 \theta_{23} \approx 0.426 \ , \ |J_{CP}| \approx 0.0254 \ ,$$

and

$$|\sin \delta| \approx 0.735 \ , \ |\sin \alpha| \approx 0.732 \ , \ |\sin \beta| \approx 1 \ \text{for} \ \theta \approx 1.437$$
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$ \cite{Ding/Zhou ('13)}
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$  

(Ding/Zhou (’13))
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$ (Ding/Zhou ('13))
Study of $\Delta(48)$ and $CP$

Angles and phases from $G_e = Z_3$, $Z = c^2$ and $X_3$  

(Ding/Zhou ('13))
Basics of leptogenesis

• baryon asymmetry of the Universe is measured well

\[ Y_B = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.77 \pm 0.24) \times 10^{-11} \]  

(WMAP ('08), Planck ('13))

• this asymmetry can be explained by decay of heavy right-handed neutrinos  

(Fukugita/Yanagida ('86))

• the three Sakharov conditions are fulfilled  

(Sakharov ('67))
Basics of leptogenesis

- baryon asymmetry of the Universe is measured well

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- this asymmetry can be explained by decay of heavy right-handed neutrinos \((\text{Fukugita/Yanagida ('86)})\)

- the three Sakharov conditions are fulfilled \((\text{Sakharov ('67)})\)
  - C and CP violation:
    - Yukawa couplings of right-handed neutrinos provide source of CP violation
Basics of leptogenesis

• baryon asymmetry of the Universe is measured well

\[ Y_B = \frac{n_B - n_{\bar{B}}}{s} \bigg|_0 = (8.77 \pm 0.24) \times 10^{-11} \]  

\textit{(WMAP ('08), Planck ('13))}

• this asymmetry can be explained by decay of heavy right-handed neutrinos  
\textit{(Fukugita/Yanagida ('86))}

• the three Sakharov conditions are fulfilled \textit{(Sakharov ('67))}
  
  • departure from thermal equilibrium:  
  Yukawa interactions of right-handed neutrinos have slow enough rate \( \Gamma < H \)
Basics of leptogenesis

• baryon asymmetry of the Universe is measured well

\[ Y_B = \frac{n_B - n\bar{B}}{s} \bigg|_0 = (8.77 \pm 0.24) \times 10^{-11} \quad (\text{WMAP (’08), Planck (’13)}) \]

• this asymmetry can be explained by decay of heavy right-handed neutrinos \((\text{Fukugita/Yanagida (’86)})\)

• the three Sakharov conditions are fulfilled \((\text{Sakharov (’67)})\)
  • baryon number violation:
    Majorana masses violate lepton number so that lepton asymmetry is generated which is partially converted into baryon asymmetry via sphaleron processes
Basics of leptogenesis

• baryon asymmetry of the Universe is measured well

\[ Y_B = \left. \frac{n_B - n_{\bar{B}}}{s} \right|_0 = (8.77 \pm 0.24) \times 10^{-11} \quad (\text{WMAP ('08), Planck ('13))} \]

• this asymmetry can be explained by decay of heavy right-handed neutrinos \( (\text{Fukugita/Yanagida ('86))} \)

• the three Sakharov conditions are fulfilled \( (\text{Sakharov ('67))} \)

• simplest scenario:

  thermal leptogenesis in which asymmetry stems from \( N_1 \) decay (with no flavour effects)
Basics of leptogenesis

• baryon asymmetry of the Universe is measured well

\[ Y_B = \frac{n_B - n_{\bar{B}}}{s} \bigg|_0 = (8.77 \pm 0.24) \times 10^{-11} \quad (\text{WMAP ('08), Planck ('13))} \]

• this asymmetry can be explained by decay of heavy right-handed neutrinos \( (\text{Fukugita/Yanagida ('86))} \)

• the three Sakharov conditions are fulfilled \( (\text{Sakharov ('67))} \)

• simplest scenario:

\[ Y_B \sim 10^{-3} \epsilon \eta \quad \text{with} \quad \epsilon \quad \text{CP asymmetry}, \quad \eta \quad \text{washout factor} \]
Basics of leptogenesis

• CP asymmetry $\epsilon$

$$\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \rightarrow Hl_\alpha) - \Gamma(N_1 \rightarrow H^*\bar{l}_\alpha)}{\Gamma(N_1 \rightarrow HL) + \Gamma(N_1 \rightarrow H^*\bar{l})}$$

• diagrammatically: the CP asymmetry arises from interference of tree-level diagram
Basics of leptogenesis

- CP asymmetry $\epsilon$

$$\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \rightarrow Hl_\alpha) - \Gamma(N_1 \rightarrow H^*\bar{l}_\alpha)}{\Gamma(N_1 \rightarrow Hl) + \Gamma(N_1 \rightarrow H^*\bar{l})}$$

- diagrammatically: the CP asymmetry arises from interference of tree-level diagram and one-loop diagrams
Basics of leptogenesis

• CP asymmetry $\epsilon$

$$\epsilon_{\alpha\alpha} = \frac{\Gamma(N_1 \rightarrow H l_\alpha) - \Gamma(N_1 \rightarrow H^* \bar{l}_\alpha)}{\Gamma(N_1 \rightarrow H l) + \Gamma(N_1 \rightarrow H^* \bar{l})}$$

• computation of $\epsilon$ in case of unflavoured leptogenesis

$$\epsilon = \frac{1}{8\pi} \sum_{j \neq 1} \frac{\text{Im} \left( (\hat{Y}_D \hat{Y}_D^\dagger)_j^2 \right)}{(\hat{Y}_D \hat{Y}_D^\dagger)_{11}} f(x_j)$$

with $\hat{Y}_D = U_R^\dagger Y_D$ and $U_R^\dagger M_R U_R^* = \text{diag}(M_1, M_2, M_3)$
Leptogenesis in flavour models

• leptogenesis has been studied in several models with $A_4$ or $S_4$ flavour symmetry

  (Jenkins/Manohar ('08), H et al. ('09), Bertuzzo et al. ('09), Aristizabal Sierra et al. ('09))

• $G_f \rightarrow G_e$ in charged lepton sector and $m_e$ is diagonal

• $G_f \rightarrow G_\nu = Z_2(\times Z_2)$ in neutrino sector and $M_R$ encodes mixing, while $Y_D$ has trivial flavour structure
Leptogenesis in flavour models

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- $G_f \rightarrow G_\nu = Z_2 \times Z_2$ in neutrino sector and $M_R$ encodes mixing, while $Y_D$ has trivial flavour structure

- for generations in 3 and $Y_D$ invariant under $G_f \epsilon$ vanishes
Leptogenesis in flavour models

- Leptogenesis has been studied in several models with $A_4$ or $S_4$ flavour symmetry
  
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- If residual $G_\nu$ is broken at level $\epsilon$,

  $\epsilon \propto \epsilon^2$ for unflavoured leptogenesis

  $[\epsilon \propto \epsilon$ for flavoured leptogenesis]
Leptogenesis in flavour models

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• if residual $G_\nu$ is broken at level $\varepsilon$,

  $$\varepsilon \propto \varepsilon^2$$

  for unflavoured leptogenesis

• if $CP$ is also a symmetry of the theory, constraints on phases and e.g. on sign of $\varepsilon$ are expected
Leptogenesis in models with flavour and \( CP \)

Consider the following scenario

- \( G_f \) & CP \( \rightarrow \) \( G_e \) in charged lepton sector and \( m_e \) is diagonal
- \( G_f \) & CP \( \rightarrow \) \( G_\nu = Z_2 \times CP \) in neutrino sector and \( M_R \) encodes mixing, while \( Y_D \) has trivial flavour structure
- assume small breaking in \( Y_D \) at level \( \varepsilon \) which is real; e.g. for our example \( G_f = S_4 \)

\[
Y_D^0 + \delta Y_D = \begin{pmatrix}
y_0 + a \varepsilon & 0 & 0 \\
0 & 0 & y_0 + b \varepsilon \\
0 & y_0 + c \varepsilon & 0
\end{pmatrix}
\]

- fit of reactor mixing angle requires \( 0.16 \lesssim \theta \lesssim 0.21 \)
Leptogenesis in models with flavour and $CP$

Result for $\epsilon$ from $N_1$ decays vs lightest neutrino mass $m_0$

$\epsilon = \lambda^4 \approx 1.6 \times 10^{-3}$; normal ordering and best fit values of $\Delta m^2_{ij}$

*(Capozzi et al. ('13)) assumed*
Leptogenesis in models with flavour and $CP$

Consider the following scenario

- $G_f \& CP \rightarrow G_e$ in charged lepton sector and $m_e$ is diagonal
- $G_f \& CP \rightarrow G_\nu = Z_2 \times CP$ in neutrino sector and $M_R$ encodes mixing, while $Y_D$ has trivial flavour structure
- assume small breaking in $Y_D$ at level $\varepsilon$ which is real; e.g. for our example $G_f = \Delta(48)$

\[
Y_D^0 + \delta Y_D = \begin{pmatrix}
  y_0 + (s + 2t) \varepsilon & 0 & 0 \\
  0 & y_0 + (s - t - \sqrt{3}u) \varepsilon & 0 \\
  0 & 0 & y_0 + (s - t + \sqrt{3}u) \varepsilon
\end{pmatrix}
\]

- fit of reactor mixing angle constrains $\theta$: $1.40 \lesssim \theta \lesssim 1.48$
Leptogenesis in models with flavour and $CP$

Result for $\epsilon$ from $N_1$ decays vs lightest neutrino mass $m_0$

$\epsilon = \lambda^4 \approx 1.6 \times 10^{-3}$; normal ordering and best fit values of $\Delta m^2_{ij}$

*(Capozzi et al. ('13)) assumed*

Notice: phases in $K_\nu$ can change sign of $\epsilon$
Leptogenesis in models with flavour and $CP$

We can understand this behaviour:

Look at $\text{Im} \left( (\hat{Y}_D \hat{Y}_D^\dagger)_{j1}^2 \right)$; for $j = 2$

$$2 (-1)^{k_1} y_0^2 \varepsilon^2 \left( -t^2 - 2tu + u^2 - \sqrt{2}(t^2 + u^2) \cos 2\theta + (t^2 - 2tu - u^2) \sin 2\theta \right) + O(\varepsilon^3)$$

and for $j = 3$

$$4 (-1)^{k_2} y_0^2 \varepsilon^2 \left( -t^2 + u^2 \right) \sin 2\theta + O(\varepsilon^3)$$
Leptogenesis in models with flavour and $CP$

We can understand this behaviour:

Now expand for $\theta = \pi/2 + \kappa$ up to $\kappa$; for $j = 2$

$$2 (-1)^{k_1} y_0^2 \varepsilon^2 \left( t^2 (-1 + \sqrt{2} - 2\kappa) + 2 t u (-1 + 2\kappa) + u^2 (1 + \sqrt{2} + 2\kappa) \right) + O(\kappa^2)$$

and for $j = 3$

$$8 (-1)^{k_2} y_0^2 \varepsilon^2 (t - u)(t + u) \kappa$$
Leptogenesis in models with flavour and $CP$

We can understand this behaviour:

Now expand for $\theta = \pi/2 + \kappa$ up to $\kappa$; for $j = 2$

$$2 (-1)^{k_1} y_0^2 \varepsilon^2 \left( t \sqrt{-1 + \sqrt{2} - 2\kappa} - u \sqrt{1 + \sqrt{2} + 2\kappa} \right)^2 + \mathcal{O}(\kappa^2)$$

and for $j = 3$

suppressed by $\kappa$

The loop function $f(x,j)$ acts as weighting factor of the different contributions.
Conclusions & outlook

• approach with flavour and CP symmetry strongly constrains lepton mixing
• results for $G_f = S_4$ or $G_f = \Delta(48)$ are encouraging
• leptogenesis can be studied in this approach
Conclusions & outlook

• continue study of different groups $G_f$ ($\Delta(3n^2)$ and $\Delta(6n^2)$) and CP:
  new mixing patterns, consistent definition of CP, ...

• explore more phenomena which involve CP phases:
  $0\nu\beta\beta$, electric dipole moments, phases of soft supersymmetry breaking terms, CKM phase, ...

Thank you for your attention.