Theoretical status of the muon $g − 2$

Andreas Nyffeler

Institute of Nuclear Physics
Johannes Gutenberg University, Mainz, Germany
nyffeler@kph.uni-mainz.de

Service de Physique Théorique
Université Libre de Bruxelles, Belgium
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Basics of the anomalous magnetic moment

Electrostatic properties of charged particles:
Charge $Q$, Magnetic moment $\vec{\mu}$, Electric dipole moment $\vec{d}$

For a spin 1/2 particle:

$$\vec{\mu} = g \frac{e}{2m} \vec{s}, \quad g = 2(1 + a), \quad a = \frac{1}{2}(g - 2): \text{anomalous magnetic moment}$$  

Dirac

Long interplay between experiment and theory: structure of fundamental forces

In Quantum Field Theory (with C,P invariance):

$$\gamma(k) = (-ie)\bar{u}(p') \begin{bmatrix} \gamma^\mu F_1(k^2) + \frac{i\sigma^{\mu\nu}k_\nu}{2m} F_2(k^2) \end{bmatrix} u(p)$$

$$F_1(0) = 1 \quad \text{and} \quad F_2(0) = a$$

$a_e$: Test of QED. Most precise determination of $\alpha = e^2/4\pi$.

$a_\mu$: Less precisely measured than $a_e$, but all sectors of Standard Model (SM), i.e. QED, Weak and QCD (hadronic), contribute significantly.

Sensitive to possible contributions from New Physics. Often (but not always!):

$$a_\ell \sim \left( \frac{m_\ell}{m_{NP}} \right)^2 \Rightarrow \left( \frac{m_\mu}{m_e} \right)^2 \sim 43000 \text{more sensitive than } a_e \text{ [exp. precision } \rightarrow \text{ factor 19]}$$
Some theoretical comments

- **Anomalous magnetic moment is finite and calculable**
  
  Corresponds to effective interaction Lagrangian of mass dimension 5:

  \[ \mathcal{L}_{\text{eff}}^{\text{AMM}} = -\frac{e_\ell a_\ell}{4m_\ell} \bar{\psi}(x) \sigma^{\mu\nu} \psi(x) F_{\mu\nu}(x) \]

  \( a_\ell = F_2(0) \) can be calculated unambiguously in renormalizable QFT, since there is no counterterm to absorb potential ultraviolet divergence.

- **Anomalous magnetic moments are dimensionless**
  
  To lowest order in perturbation theory in quantum electrodynamics (QED):

  \[ a_e = a_\mu = \frac{\alpha}{2\pi} \]

  [Schwinger ’47/’48]

- **Loops with different masses ⇒ \( a_e \neq a_\mu \)**
  
  - Internal large masses decouple (not always!):

    \[ = \left[ \frac{1}{45} \left( \frac{m_e}{m_\mu} \right)^2 + \mathcal{O} \left( \frac{m_\mu^4}{m_e^4} \ln \frac{m_\mu}{m_e} \right) \right] \left( \frac{\alpha}{\pi} \right)^2 \]

  - Internal small masses give rise to large log’s of mass ratios:

    \[ = \left[ \frac{1}{3} \ln \frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O} \left( \frac{m_e}{m_\mu} \right) \right] \left( \frac{\alpha}{\pi} \right)^2 \]
Electron $g - 2$
Main contribution in Standard Model (SM) from mass-independent Feynman diagrams in QED with electrons in internal lines (perturbative series in $\alpha$):

$$a_e^{SM} = \sum_{n=1}^{5} c_n \left( \frac{\alpha}{\pi} \right)^n + 2.7478(2) \times 10^{-12} \ [\text{Loops in QED with } \mu, \tau] + 0.0297(5) \times 10^{-12} \ [\text{weak interactions}] + 1.682(20) \times 10^{-12} \ [\text{strong interactions / hadrons}]$$

The numbers are based on the paper by Aoyama et al. 2012.
QED: mass-independent contributions to $a_e$

- **$\alpha$: 1-loop**, 1 Feynman diagram; Schwinger '47/'48:
  
  \[ c_1 = \frac{1}{2} \]

- **$\alpha^2$: 2-loops**, 7 Feynman diagrams; Petermann '57, Sommerfield '57:
  
  \[ c_2 = \frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4} \zeta(3) = -0.32847896557919378 \ldots \]

- **$\alpha^3$: 3-loops**, 72 Feynman diagrams; \ldots, Laporta, Remiddi '96:
  
  \[
  c_3 = \frac{28259}{5184} + \frac{17101}{810} \pi^2 - \frac{298}{9} \pi^2 \ln 2 + \frac{139}{18} \zeta(3) - \frac{239}{2160} \pi^4 \\
  + \frac{83}{72} \pi^2 \zeta(3) - \frac{215}{24} \zeta(5) + \frac{100}{3} \left\{ \text{Li}_4 \left( \frac{1}{2} \right) + \frac{1}{24} \ln^4 2 - \frac{1}{24} \pi^2 \ln^2 2 \right\} \\
  = 1.181241456587 \ldots
  \]

- **$\alpha^4$: 4-loops**, 891 Feynman diagrams; Kinoshita et al. '99, \ldots, Aoyama et al. '08; '12:
  
  \[ c_4 = -1.9106(20) \text{ (numerical evaluation)} \]

- **$\alpha^5$: 5-loops**, 12672 Feynman diagrams; Aoyama et al. '05, \ldots, '12:
  
  \[ c_5 = 9.16(58) \text{ (numerical evaluation)} \]

Replaces earlier rough estimate $c_5 = 0.0 \pm 4.6$.

Result removes biggest theoretical uncertainty in $a_e$!
Mass-independent 2-loop Feynman diagrams in $a_e$
Mass-independent 3-loop Feynman diagrams in $a_e$
Electron $g - 2$: Experiment

Latest experiment: Hanneke, Fogwell, Gabrielse, 2008

![Cylindrical Penning trap for single electron](image)

(Cylindrical Penning trap for single electron
(1-electron quantum cyclotron)

Source: Hanneke et al.

\[
\frac{g_e}{2} = \frac{\nu_s}{\nu_c} \approx 1 + \frac{\bar{\nu} - \bar{\nu}^2}{\bar{f}_c + 3\delta/2 + \bar{\nu}_z^2/(2\bar{f}_c)} + \frac{\Delta g_{\text{cav}}}{2}
\]

$\nu_s$ = spin precession frequency; $\nu_c, \bar{\nu}_c$ = cyclotron frequency: free electron, electron in Penning trap; $\delta/\nu_c = h\nu_c/(m_e c^2) \approx 10^{-9}$ = relativistic correction

4 quantities are measured precisely in experiment:

\(\bar{f}_c = \bar{\nu}_c - \frac{3}{2}\delta \approx 149 \text{ GHz}; \)
\(\bar{\nu}_a = \frac{g}{2}\nu_c - \bar{\nu}_c \approx 173 \text{ MHz};\)
\(\bar{\nu}_z \approx 200 \text{ MHz} = \text{oscillation frequency in axial direction};\)
\(\Delta g_{\text{cav}} = \text{corrections due to oscillation modes in cavity}\)

\[
\Rightarrow a_e^{\text{exp}} = 0.00115965218073(28) \quad [0.24 \text{ ppb} \approx 1 \text{ part in 4 billions}]
\]

[\text{Kusch & Foley, 1947/48: 4\% precision}]

Precision in $g_e/2$ even 0.28 ppt $\approx 1$ part in 4 trillions!
Determination of fine-structure constant $\alpha$ from $g - 2$ of electron

- Recent measurement of $\alpha$ via recoil-velocity of Rubidium atoms in atom interferometer (Bouchendira et al. 2011):
  \[
  \alpha^{-1}(\text{Rb}) = 137.035\text{ 999 037(91)} \quad [0.66\text{ppb}]
  \]
  This leads to (Aoyama et al. 2012):
  \[
  a^\text{SM}_e(\text{Rb}) = 1\text{ 159 652 181.82 (6) (4) (2) (78) [78]} \times 10^{-12} \quad [0.67\text{ppb}]
  \]
  \[
  \Rightarrow a^\text{exp}_e - a^\text{SM}_e(\text{Rb}) = -1.09(0.83) \times 10^{-12} \quad \text{[Error from $\alpha(\text{Rb})$ dominates!]}\]
  \rightarrow \text{Test of QED!}

- Use $a^\text{exp}_e$ to determine $\alpha$ from series expansion in QED (contributions from weak and strong interactions under control!). Assume: Standard Model “correct”, no New Physics (Aoyama et al. 2012):
  \[
  \alpha^{-1}(a_e) = 137.035\text{ 999 1657 (68) (46) (24) (331) [342]} \quad [0.25\text{ppb}]
  \]
  The uncertainty from theory has been improved by a factor 4.5 by Aoyama et al. 2012, the experimental uncertainty in $a^\text{exp}_e$ is now the limiting factor.

- Today the most precise determination of the fine-structure constant $\alpha$, a fundamental parameter of the Standard Model.
Muon $g - 2$
The Brookhaven Muon $g - 2$ Experiment

The first measurements of the anomalous magnetic moment of the muon were performed in 1960 at CERN, $a_{\mu}^{\text{exp}} = 0.00113(14)$ (Garwin et al.) [12% precision] and improved until 1979: $a_{\mu}^{\text{exp}} = 0.001165924(85)$ [7 ppm] (Bailey et al.)

In 1997, a new experiment started at the Brookhaven National Laboratory (BNL):

Angular frequencies for cyclotron precession $\omega_c$ and spin precession $\omega_s$:

$$\omega_c = \frac{eB}{m_\mu \gamma}, \quad \omega_s = \frac{eB}{m_\mu \gamma} + a_\mu \frac{eB}{m_\mu}, \quad \omega_a = a_\mu \frac{eB}{m_\mu}$$

$$\gamma = 1/\sqrt{1-(v/c)^2}.$$ With an electric field to focus the muon beam one gets:

$$\tilde{\omega}_a = \frac{e}{m_\mu} \left( a_\mu \vec{B} - \left[ a_\mu - \frac{1}{\gamma^2 - 1} \right] \vec{v} \times \vec{E} \right)$$

Term with $\vec{E}$ drops out, if $\gamma = \sqrt{1 + 1/a_\mu} = 29.3$: "magic $\gamma$" $\rightarrow p_\mu = 3.094$ GeV/c
The Brookhaven Muon $g - 2$ Experiment: storage ring
The Brookhaven Muon $g - 2$ Experiment: determination of $a_\mu$

Histogram with 3.6 billion decays of $\mu^-$:

$$N(t) = N_0(E) \exp\left(\frac{-t}{\gamma \tau_\mu}\right) \times [1 + A(E) \sin(\omega_a t + \phi(E))]$$

Exponential decay with mean lifetime:
$$\tau_{\mu,\text{lab}} = \gamma \tau_\mu \approx 64.378 \mu s$$
(in lab system).

Oscillations due to angular frequency
$$\omega_a = a_\mu eB/m_\mu.$$  

$$a_\mu = \frac{R}{\lambda - R} \text{ where } R = \frac{\omega_a}{\omega_p} \text{ and } \lambda = \frac{\mu \mu}{\mu_p}$$

Brookhaven experiment measures $\omega_a$ and $\omega_p$ (spin precession frequency for proton).

$\lambda$ from hyperfine splitting of muonium $(\mu^+ e^-)$ (external input).
### Milestones in measurements of $a_\mu$

<table>
<thead>
<tr>
<th>Authors</th>
<th>Lab</th>
<th>Muon Anomaly</th>
</tr>
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<tbody>
<tr>
<td>Garwin et al. '60</td>
<td>CERN</td>
<td>0.001 13(14)</td>
</tr>
<tr>
<td>Charpak et al. '61</td>
<td>CERN</td>
<td>0.001 145(22)</td>
</tr>
<tr>
<td>Charpak et al. '62</td>
<td>CERN</td>
<td>0.001 162(5)</td>
</tr>
<tr>
<td>Farley et al. '66</td>
<td>CERN</td>
<td>0.001 165(3)</td>
</tr>
<tr>
<td>Bailey et al. '68</td>
<td>CERN</td>
<td>0.001 166 16(31)</td>
</tr>
<tr>
<td>Bailey et al. '79</td>
<td>CERN</td>
<td>0.001 165 923 0(84)</td>
</tr>
<tr>
<td>Brown et al. '00</td>
<td>BNL</td>
<td>0.001 165 919 1(59) ($\mu^+$)</td>
</tr>
<tr>
<td>Brown et al. '01</td>
<td>BNL</td>
<td>0.001 165 920 2(14)(6) ($\mu^+$)</td>
</tr>
<tr>
<td>Bennett et al. '02</td>
<td>BNL</td>
<td>0.001 165 920 4(7)(5) ($\mu^+$)</td>
</tr>
<tr>
<td>Bennett et al. '04</td>
<td>BNL</td>
<td>0.001 165 921 4(8)(3) ($\mu^-$)</td>
</tr>
</tbody>
</table>

World average experimental value (dominated by $g - 2$ Collaboration at BNL, Bennett et al. '06 + CODATA 2008 value for $\lambda = \mu_\mu/\mu_p$):

$$a_\mu^{\text{exp}} = (116\ 592\ 089 \pm 63) \times 10^{-11} \quad [0.5\text{ppm}]$$

Goal of new planned $g - 2$ experiments: $\delta a_\mu = 16 \times 10^{-11}$

**Fermilab E989**: partly recycled from BNL: moved ring magnet! (http://muon-g-2.fnal.gov/bigmove/) First beam in 2017, should reach this precision by 2020 (?). **J-PARC E24**: completely new concept with low-energy muons, not magic $\gamma$. Aims in Phase 1 for about $\delta a_\mu = 45 \times 10^{-11}$ by 2019.

**Theory needs to match this precision!**

For comparison: Electron (stable !) (Hanneke et al. '08):

$$a_e^{\text{exp}} = (1\ 159\ 652\ 180.73 \pm 0.28) \times 10^{-12} \quad [0.24\text{ppb}]$$
Muon $g - 2$: Theory

In Standard Model (SM):

$$a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{weak}} + a_{\mu}^{\text{had}}$$

In contrast to $a_e$, here now the contributions from weak and strong interactions (hadrons) are relevant, since $a_{\mu} \sim (m_{\mu}/M)^2$.

QED contributions

- Diagrams with internal electron loops are enhanced.
- At 2-loops: vacuum polarization from electron loops enhanced by QED short-distance logarithm
- At 3-loops: light-by-light scattering from electron loops enhanced by QED infrared logarithm [Aldins et al. '69, '70; Laporta, Remiddi '93]

$$a_{\mu}^{(3)} \Bigg|_{\text{lbyl}} = \left[ \frac{2}{3} \pi^2 \ln \frac{m_{\mu}}{m_e} + \ldots \right] \left( \frac{\alpha}{\pi} \right)^3 = 20.947 \ldots \left( \frac{\alpha}{\pi} \right)^3$$

- Loops with tau’s suppressed (decoupling)
QED result up to 5 loops

Include contributions from all leptons (Aoyama et al. '12):

\[
a_{\mu}^{\text{QED}} = 0.5 \times \left( \frac{\alpha}{\pi} \right) + 0.765 \, 857 \, 425 \times \left( \frac{\alpha}{\pi} \right)^2 \frac{m_\mu/m_{e,\tau}}{\text{num. int.}}
\]

\[
+ 24.050 \, 509 \, 96 \times \left( \frac{\alpha}{\pi} \right)^3 + 130.8796 \times \left( \frac{\alpha}{\pi} \right)^4
\]

\[
+ 753.29 \times \left( \frac{\alpha}{\pi} \right)^5 \frac{\text{num. int.}}{\text{num. int.}}
\]

\[
= 116 \, 584 \, 718.853 \times 10^{-11}
\]

- Earlier evaluation of 5-loop contribution yielded \( c_5 = 662(20) \) (Kinoshita, Nio '06, numerical evaluation of 2958 diagrams, known or likely to be enhanced). New value is 4.5\( \sigma \) from this leading log estimate and 20 times more precise.

- Aoyama et al. '12: What about the 6-loop term? Leading contribution from light-by-light scattering with electron loop and insertions of vacuum-polarization loops of electrons into each photon line \( \Rightarrow a_{\mu}^{\text{QED}}(6\text{-loops}) \sim 0.1 \times 10^{-11} \)
Contributions from weak interaction

Numbers from recent reanalysis by Gnendiger et al. '13.

1-loop contributions [Jackiw + Weinberg, 1972; ...]:

\[ a_{\mu}^{\text{weak}, (1)}(W) = \frac{\sqrt{2}G_\mu m_\mu^2}{16\pi^2} \frac{10}{3} + \mathcal{O}(m_\mu^2/M_W^2) = 388.70 \times 10^{-11} \]

\[ a_{\mu}^{\text{weak}, (1)}(Z) = \frac{\sqrt{2}G_\mu m_\mu^2}{16\pi^2} \frac{(-1 + 4s_W^2)^2 - 5}{3} + \mathcal{O}(m_\mu^2/M_Z^2) = -193.89 \times 10^{-11} \]

Contribution from Higgs negligible: \( a_{\mu}^{\text{weak}, (1)}(H) \leq 5 \times 10^{-14} \) for \( m_H \geq 114 \) GeV.

\[ a_{\mu}^{\text{weak}, (1)} = (194.80 \pm 0.01) \times 10^{-11} \]

2-loop contributions (1678 diagrams) [Czarnecki et al. '95, '96; ...]:

\[ a_{\mu}^{\text{weak}, (2)} = (-41.2 \pm 1.0) \times 10^{-11}, \text{ large since } \sim G_F m_\mu^2 \frac{\alpha}{\pi} \ln \frac{M_Z}{m_\mu} \]

Total weak contribution:

\[ a_{\mu}^{\text{weak}} = (153.6 \pm 1.0) \times 10^{-11} \]

Under control! With knowledge of \( M_H = 125.6 \pm 1.5 \) GeV, uncertainty now mostly hadronic \( \pm 1.0 \times 10^{-11} \) (Peris et al. '95; Knecht et al. '02; Czarnecki et al. '03, '06). 3-loop effects via RG: \( \pm 0.20 \times 10^{-11} \) (Degrassi, Giudice '98; Czarnecki et al. '03).
Hadronic contributions to $g - 2$
The strong interactions (Quantum Chromodynamics)

- Strong interactions: quantum chromodynamics (QCD) with quarks and gluons
- Observed particles in Nature: Hadrons
  1. Mesons (quark + antiquark: \(q\bar{q}\)): \(\pi, K, \eta, \rho, \ldots\)
  2. Baryons (3 quarks: \(qqq\)): \(p, n, \Lambda, \Sigma, \Delta, \ldots\)

- Cannot describe hadrons in series expansion in strong coupling constant of QCD with \(\alpha_s(E = m_{\text{proton}}) \approx 0.5\).
  Particularly true for light hadrons which consist of three lightest quarks \(u, d, s\). Non-perturbative effects like “confinement” of quarks and gluons inside hadrons.

Possible approaches to QCD at low energies:
  1. Lattice QCD: limited applications, often still limited precision
  2. Effective quantum field theories with hadrons (chiral perturbation theory): limited validity
  3. Simplifying hadronic models: model uncertainties not controllable
  4. Dispersion relations: extend validity of EFT’s, reduce model dependence, often not all the needed input data available
Hadronic contributions to the muon $g - 2$

Largest source of uncertainty in theoretical prediction of $a_\mu$!

Different types of contributions:

(a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3), O(\alpha^4)$

(b) Hadronic light-by-light scattering $O(\alpha^3), O(\alpha^4)$

(c) 2-loop electroweak contributions $O(\alpha G_F m_\mu^2)$

2-Loop EW
Small hadronic uncertainty from triangle diagrams.
Anomaly cancellation within each generation!
Cannot separate leptons and quarks!
Hadronic vacuum polarization (HVP)
Hadronic vacuum polarization

Optical theorem (from unitarity; conservation of probability) for hadronic contribution → dispersion relation:

\[ \text{Im} \sim \left( \frac{\alpha}{\pi} \right)^2 \int_0^\infty \frac{ds}{s} K(s) R(s), \quad R(s) = \frac{\sigma(e^+ e^- \rightarrow \gamma^* \rightarrow \text{hadrons})}{\sigma(e^+ e^- \rightarrow \gamma^* \rightarrow \mu^+ \mu^-)} \]

[Bouchiat, Michel '61; Durand '62; Brodsky, de Rafael '68; Gourdin, de Rafael '69]

\( K(s) \) slowly varying, positive function \( \Rightarrow a_{\mu}^{\text{HVP}} \) positive. Data for hadronic cross section \( \sigma \) at low center-of-mass energies \( \sqrt{s} \) important due to factor \( 1/s \): \( \sim 70\% \) from \( \pi\pi \) \([\rho(770)]\) channel, \( \sim 90\% \) from energy region below 1.8 GeV.

Other method instead of energy scan: "Radiative return" at colliders with fixed center-of-mass energy (DAΦNE, B-Factories, BEPC) [Binner et al. '99; Czyż et al. '00-'03]
Measured hadronic cross-section

Pion form factor $|F_\pi(E)|^2$ (
\begin{align*}
R(s) &= \frac{1}{4} \left(1 - \frac{4m_\pi^2}{s}\right)^3 |F_\pi(s)|^2 \\
&\quad (4m_\pi^2 < s < 9m_\pi^2)
\end{align*}

R-ratio:

Jegerlehner, AN '09
### Hadronic vacuum polarization: some recent evaluations

<table>
<thead>
<tr>
<th>Authors</th>
<th>Contribution to $a^{\text{HVP}}_\mu \times 10^{11}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jegerlehner '08; Jegerlehner, AN '09 ($e^+e^-$)</td>
<td>$6903.0 \pm 52.6$</td>
</tr>
<tr>
<td>Davier et al. '09 ($e^+e^-$) [+ $\tau$]</td>
<td>$6955 \pm 41$ [7053 ± 45]</td>
</tr>
<tr>
<td>Teubner et al. '09 ($e^+e^-$)</td>
<td>$6894 \pm 40$</td>
</tr>
<tr>
<td>Davier et al. '11, '14 ($e^+e^-$) [+ $\tau$]</td>
<td>$6923 \pm 42$ [7030 ± 44]</td>
</tr>
<tr>
<td>Jegerlehner, Szafron '11 ($e^+e^-$) [+ $\tau$]</td>
<td>$6907.5 \pm 47.2$ [6909.6 ± 46.5]</td>
</tr>
<tr>
<td>Hagiwara et al. '11 ($e^+e^-$)</td>
<td>$6949.1 \pm 42.7$</td>
</tr>
<tr>
<td>Benayoun et al. '15 ($e^+e^- + \tau$: BHLS improved)</td>
<td>$6818.6 \pm 32.0$</td>
</tr>
<tr>
<td>Jegerlehner '15 ($e^+e^-$) [+ $\tau$]</td>
<td>$6885.7 \pm 42.8$ [6889.1 ± 35.2]</td>
</tr>
</tbody>
</table>

- **Precision:** < 1%. Non-trivial because of radiative corrections (radiated photons).
- Even if values for $a^{\text{HVP}}_\mu$ after integration agree quite well, the systematic differences of a few % in the shape of the spectral functions from different experiments (BABAR, BES III, CMD-2, KLOE, SND) indicate that we do not yet have a complete understanding.
- **Use of $\tau$ data:** additional sources of isospin violation? Ghozzi, Jegerlehner '04; Benayoun et al. '08, '09; Wolfe, Maltman '09; Jegerlehner, Szafron '11 ($\rho - \gamma$-mixing), also included Jegerlehner '15 and in BHLS-approach by Benayoun et al. '15 (additional BHLS model uncertainty can lead to maximal shift in central value of $^{+15}_{-27} \times 10^{-11}$).
- **Lattice QCD:** Various groups are working on it, precision at level of 5-10%, not yet competitive with phenomenological evaluations.
Hadronic light-by-light scattering (HLbL)
Hadronic light-by-light scattering in the muon $g - 2$

**QED:** light-by-light scattering at higher orders in perturbation series via lepton-loop:

\[
\begin{align*}
\gamma & \quad \gamma \quad \gamma \\
\gamma & \quad e \quad \gamma
\end{align*}
\]

In muon $g - 2$:

\[
\Rightarrow
\]

Hadronic light-by-light scattering in muon $g - 2$ from strong interactions (QCD):

\[
\begin{align*}
\mu & \quad \gamma \\
\mu & \quad =
\end{align*}
\]

\[
\begin{align*}
\pi^0, \eta, \eta' & \quad + \ldots & \quad + \ldots & \quad \pi^0
\end{align*}
\]

Coupling of photons to hadrons, e.g. $\pi^0$, via form factor:

\[
\begin{align*}
\gamma & \quad \gamma
\end{align*}
\]

View before 2014: in contrast to HVP, no direct relation to experimental data $\rightarrow$ size and even sign of contribution to $a_\mu$ unknown!

Approach: use hadronic model at low energies with exchanges and loops of resonances and some (dressed) “quark-loop” at high energies.

**Problems:** Four-point function depends on several invariant momenta $\Rightarrow$ distinction between low and high energies not as easy as for two-point function in HVP.

**Mixed regions:** one loop momentum $Q_1^2$ large, the other $Q_2^2$ small and vice versa.
HLbL in muon $g - 2$

- Only model calculations so far: large uncertainties, difficult to control.
- Frequently used estimates:

\[
\begin{align*}
    a_{\mu}^{\text{HLbL}} &= (105 \pm 26) \times 10^{-11} \quad \text{(Prades, de Rafael, Vainshtein '09)} \\
    a_{\mu}^{\text{HLbL}} &= (116 \pm 40) \times 10^{-11} \quad \text{(AN '09; Jegerlehner, AN '09)}
\end{align*}
\]

Based almost on same input: calculations by various groups using different models for individual contributions. Error estimates are mostly guesses!

- Need much better understanding of complicated hadronic dynamics to get reliable error estimate of $\pm 20 \times 10^{-11}$ ($\delta a_{\mu}^{\text{future exp}} = 16 \times 10^{-11}$).
- Recent new proposal: Colangelo et al. '14, '15; Pauk, Vanderhaeghen '14: use dispersion relations (DR) to connect contribution to HLbL from presumably numerically dominant light pseudoscalars to in principle measurable form factors and cross-sections:

\[
\begin{align*}
    \gamma^* \gamma^* &\rightarrow \pi^0, \eta, \eta' \\
    \gamma^* \gamma^* &\rightarrow \pi^+ \pi^-, \pi^0 \pi^0
\end{align*}
\]

Could connect HLbL uncertainty to exp. measurement errors, like HVP.

- Maybe in future: HLbL from Lattice QCD. First steps: Blum et al. '05, . . . , '14, '15. Work ongoing by Mainz group: Green et al. '15.
Approach to HLbL in muon $g - 2$ up to now

Classification of de Rafael '94

Chiral counting $p^2$ (from Chiral Perturbation Theory (ChPT)) and large-$N_C$ counting as guideline to classify contributions (all higher orders in $p^2$ and $N_C$ contribute):

\[
\begin{align*}
\mu^- (p) & \quad \mu^- (p') \\
= & \quad \pi^+ (\rho) \\
+ & \quad \pi^0, \eta, \eta' \\
+ & \quad \rho \\
+ & \quad f_0, a_1, f_2, \ldots
\end{align*}
\]

Chiral counting: $p^4$

$N_C$-counting: $1$

pion-loop (dressed)

pseudoscalar exchanges

Exchange of other resonances ($f_0, a_1, f_2 \ldots$)

quark-loop (dressed)

Relevant scales in HLbL ($\langle VVVV \rangle$ with off-shell photons !): $0 - 2 \text{ GeV} \gg m_\mu$

Constrain models using experimental data (processes of hadrons with photons: decays, form factors, scattering) and theory (ChPT at low energies; short-distance constraints from pQCD / OPE at high momenta).

General analysis of four-point function $\Pi_{\mu\nu\rho\sigma}(q_1, q_2, q_3)$ relevant for $g - 2$: Bijnens et al. '96; Bijnens (Talk at $g - 2$ Workshop, Mainz, '14); Eichmann et al. '14, '15; Colangelo et al. '15.
HLbL scattering: summary of selected results

\[ k = p' - p \]

Chiral counting: \[ p^4 \]

\( N_C \)-counting: \[ 1 \]

Contribution to \( a_\mu \times 10^{11} \):

<table>
<thead>
<tr>
<th>Model</th>
<th>( +b )</th>
<th>( -b )</th>
<th>( +b )</th>
<th>( -b )</th>
<th>( +b )</th>
<th>( -b )</th>
<th>( +b )</th>
<th>( -b )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BPP</td>
<td>+83 (32)</td>
<td>-19 (13)</td>
<td>+85 (13)</td>
<td>-4 (3)</td>
<td>+21 (3)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>HKS</td>
<td>+90 (15)</td>
<td>-5 (8)</td>
<td>+83 (6)</td>
<td>+1.7 (1.7)</td>
<td>+10 (11)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>KN</td>
<td>+80 (40)</td>
<td>0 (10)</td>
<td>+83 (12)</td>
<td>+22 (5)</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MV</td>
<td>+136 (25)</td>
<td>0 (10)</td>
<td>+114 (10)</td>
<td>+8 (12)</td>
<td>2.3 [c-quark]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2007:</td>
<td>+110 (40)</td>
<td>-19 (19)</td>
<td>+114 (13)</td>
<td>+8 (12)</td>
<td>2.3 [c-quark]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>PdRV:</td>
<td>+105 (26)</td>
<td>-19 (19)</td>
<td>+114 (13)</td>
<td>+8 (12)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N,JN:</td>
<td>+116 (40)</td>
<td>-19 (13)</td>
<td>+99 (16)</td>
<td>+8 (12)</td>
<td>2.3 [c-quark]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

ud.: -45 ud.: +\infty \quad \text{ud.: +60}

ud. = undressed, i.e. point vertices without form factors

BPP = Bijnens, Pallante, Prades '96, '02; HKS = Hayakawa, Kinoshita, Sanda '96, '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; 2007 = Bijnens, Prades; Miller, de Rafael, Roberts; PdRV = Prades, de Rafael, Vainshtein '09 (compilation; “Glasgow consensus”); N,JN = AN '09; Jegerlehner, AN '09 (compilation)

Pseudoscalars: numerically dominant contribution (according to most models !).

Recall (in units of \( 10^{-11} \)):
\[ \delta a_\mu (\text{HVP}) \approx 45; \quad \delta a_\mu (\text{exp [BNL]}) = 63; \quad \delta a_\mu (\text{future exp}) = 16 \]
Data-driven approach to HLbL using dispersion relations (DR)

Strategy: Split contributions to HLbL into two parts:

I: **Data-driven evaluation using DR** (hopefully numerically dominant):
   1. $\pi^0, \eta, \eta'$ poles
   2. $\pi\pi$ intermediate state

II: **Model dependent evaluation** (hopefully numerically subdominant):
   1. Axial vectors ($3\pi$-intermediate state), ...
   2. Quark-loop, matching with pQCD

Error goals: Part I: 10% precision (data-driven), Part II: 30% precision.
To achieve overall error of about 20% ($\delta a^{\overline{\text{HLbL}}}_\mu = 20 \times 10^{-11}$).

Colangelo et al. '14; '15:
Classify intermediate states in four-point function, then project onto $g-2$.

\[
\Pi_{\mu\nu\lambda\sigma} = \Pi_{\mu\nu\lambda\sigma}^{\pi^0} + \Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}} + \Pi_{\mu\nu\lambda\sigma}^{\pi\pi} + \cdots
\]

$\Pi_{\mu\nu\lambda\sigma}^{\pi^0}$ = pion pole (similarly for $\eta, \eta'$)
$\Pi_{\mu\nu\lambda\sigma}^{\text{FsQED}}$ = scalar QED with vertices dressed by pion vector form factor $F^V_\pi$
$\Pi_{\mu\nu\lambda\sigma}^{\pi\pi}$ = remaining $\pi\pi$ contribution

Pauk, Vanderhaeghen '14: Write DR directly for Pauli form factor $F_2(k^2)$. 

---

![Diagrams](attachment://diagrams.png)
\( a_\mu^{\text{HLbL}, \mathcal{P}}; \mathcal{P} = \pi^0, \eta, \eta' \): impact of precision of form factor measurements

AN: work in preparation

In Jegerlehner, AN '09, a 3-dimensional integral representation for the pseudoscalar-pole contribution was derived. Schematically:

\[
a_\mu^{\text{HLbL}, \mathcal{P}} = \int_0^{\infty} dQ_1 \int_0^{\infty} dQ_2 \int_{-1}^1 d\tau \sum_i w_i(Q_1, Q_2, \tau) f_i(Q_1, Q_2, \tau)
\]

with universal weight functions \( w_i \) (for Euclidean (space-like) momenta: \( Q_1 \cdot Q_2 = |Q_1||Q_2|\tau, \tau = \cos \theta \)). Dependence on form factors resides in the \( f_i \).

Weight functions \( w_i \):

- Relevant momentum regions below 1 GeV for \( \pi^0 \), below 1.5 GeV for \( \eta, \eta' \).

- Analysis of current and future measurement precision of single-virtual \( \mathcal{F}_\mathcal{P}\gamma^*\gamma^*(-Q^2, 0) \) and double-virtual transition form factor \( \mathcal{F}_\mathcal{P}\gamma^*\gamma^*(-Q^2_1, -Q^2_2) \), based on Monte Carlo study for BES III by Denig, Redmer, Wasser.

- Data-driven precision for HLbL pseudoscalar-pole contribution that could be achieved in a few years:

\[
\frac{\delta a_\mu^{\text{HLbL}, \pi^0}}{a_\mu^{\text{HLbL}, \pi^0}} = 14\%
\]
\[
\frac{\delta a_\mu^{\text{HLbL}, \eta}}{a_\mu^{\text{HLbL}, \eta}} = 23\%
\]
\[
\frac{\delta a_\mu^{\text{HLbL}, \eta'}}{a_\mu^{\text{HLbL}, \eta'}} = 15\%
\]

Top: weight functions \( w_{1,2}(Q_1, Q_2, \tau) \) for \( \pi^0 \) with \( \theta = 90^\circ (\tau = 0) \).

Bottom: weight functions \( w_1(Q_1, Q_2, \tau) \) for \( \eta \) (left) and \( \eta' \) (right).
Muon $g - 2$: current status

<table>
<thead>
<tr>
<th>Contribution</th>
<th>$a_\mu \times 10^{11}$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>QED (leptons)</td>
<td>116 584 718.853 ± 0.036</td>
<td>Aoyama et al. '12</td>
</tr>
<tr>
<td>Electroweak</td>
<td>153.6 ± 1.0</td>
<td>Gnendiger et al. '13</td>
</tr>
<tr>
<td>HVP: LO</td>
<td>6907.5 ± 47.2</td>
<td>Jegerlehner, Szafron '11</td>
</tr>
<tr>
<td>NLO</td>
<td>-100.3 ± 2.2</td>
<td>Jegerlehner, Szafron '11</td>
</tr>
<tr>
<td>NNLO</td>
<td>12.4 ± 0.1</td>
<td>Kurz et al. '14</td>
</tr>
<tr>
<td>HLbL</td>
<td>116 ± 40</td>
<td>Jegerlehner, AN '09</td>
</tr>
<tr>
<td>NLO</td>
<td>3 ± 2</td>
<td>Colangelo et al. '14</td>
</tr>
<tr>
<td>Theory (SM)</td>
<td>116 591 811 ± 62</td>
<td>Bennett et al. '06</td>
</tr>
<tr>
<td>Experiment</td>
<td>116 592 089 ± 63</td>
<td></td>
</tr>
<tr>
<td>Experiment - Theory</td>
<td>278 ± 88</td>
<td>3.1 σ</td>
</tr>
</tbody>
</table>

HVP: Hadronic vacuum polarization
HLbL: Hadronic light-by-light scattering
Other estimate: $a_\mu^{HLbL} = (105 \pm 26) \times 10^{-11}$ (Prades, de Rafael, Vainshtein '09).

Discrepancy a sign of New Physics?

Hadronic uncertainties need to be better controlled in order to fully profit from future $g - 2$ experiments at Fermilab and J-PARC with $\delta a_\mu = 16 \times 10^{-11}$.

Way forward for HVP seems clear: more precise measurements for $\sigma(e^+e^- \rightarrow \text{hadrons})$. Not so obvious how to improve HLbL.
Muon $g - 2$: other recent evaluations

$\alpha_\mu \times 10^{10} - 11659000$

Source: Hagiwara et al. '11. Note units of $10^{-10}$!

Aoyama et al. '12: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (249 \pm 87) \times 10^{-11}$ [2.9 $\sigma$]

Benayoun et al. '15: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (376.8 \pm 75.3) \times 10^{-11}$ [5.0 $\sigma$]

Jegerlehner '15: $a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (310 \pm 82) \times 10^{-11}$ [3.8 $\sigma$]
New Physics contributions to the muon $g - 2$
Tests of the Standard Model and search for New Physics

- Standard Model (SM) of particle physics very successful in precise description of a huge amount of experimental data, with a few exceptions (3 – 4 standard deviations).

- Some experimental facts (neutrino masses, baryon asymmetry in the universe, dark matter) and some theoretical arguments, which point to New Physics beyond the Standard Model.

- There are several indications that new particles (forces) should show up in the mass range 100 GeV – 1 TeV.
Tests of the Standard Model and search for New Physics (continued)

Search for New Physics with two complementary approaches:

1. **High Energy Physics:**
   - e.g. Large Hadron Collider (LHC) at CERN
   - Direct production of new particles
   - e.g. heavy $Z'$ $\Rightarrow$ resonance peak in invariant mass distribution of $\mu^+\mu^-$ at $M_{Z'}$.

2. **Precision physics:**
   - e.g. anomalous magnetic moments $a_e, a_\mu$
   - Indirect effects of virtual particles in quantum corrections
   $\Rightarrow$ Deviations from precise predictions in SM
   
   \[ a_\ell \sim \left( \frac{m_\ell}{M_{Z'}} \right)^2 \]

   Note: there are also non-decoupling contributions of heavy New Physics!
   
   Another example: new light vector meson ("dark photon") with $M_{\gamma'} \sim (10 - 100)$ MeV.

$a_e, a_\mu$ allow to exclude some models of New Physics or to constrain their parameter space.
New Physics contributions to the muon $g - 2$

Define:

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (290 \pm 90) \times 10^{-11} \quad \text{(Jegerlehner, AN '09)}$$

Absolute size of discrepancy is actually unexpectedly large, compared to weak contribution (although there is some cancellation there):

$$a_\mu^{\text{weak}} = a_\mu^{\text{weak, (1)}}(W) + a_\mu^{\text{weak, (1)}}(Z) + a_\mu^{\text{weak, (2)}}$$

$$= (389 - 194 - 41) \times 10^{-11}$$

$$= 154 \times 10^{-11}$$

Assume that New Physics contribution with $M_{\text{NP}} \gg m_\mu$ decouples:

$$a_\mu^{\text{NP}} = C \frac{m_\mu^2}{M_{\text{NP}}^2}$$

where naturally $C = \frac{\alpha}{\pi}$, like from a one-loop QED diagram, but with new particles. Typical New Physics scales required to satisfy $a_\mu^{\text{NP}} = \Delta a_\mu$:

<table>
<thead>
<tr>
<th>$M_{\text{NP}}$</th>
<th>$C$</th>
<th>$\frac{\alpha}{\pi}$</th>
<th>$(\frac{\alpha}{\pi})^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0$^{+0.4}_{-0.3}$ TeV</td>
<td>1</td>
<td>$\frac{\alpha}{\pi}$</td>
<td>$5^{+1}_{-1}$ GeV</td>
</tr>
<tr>
<td>100$^{+21}_{-13}$ GeV</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Therefore, for New Physics model with particles in 250 – 300 GeV mass range and electroweak-size couplings $O(\alpha)$, we need some additional enhancement factor, like large $\tan \beta$ in the MSSM, to explain the discrepancy $\Delta a_\mu$. 
Generic 1-loop New Physics contributions to $g - 2$

Possible New Physics contributions:

a) \[
\begin{array}{c}
\gamma \\
m_\mu \\
M_0[S,P] \\
m_\mu \\
M
\end{array}
\]

b) \[
\begin{array}{c}
\gamma \\
M \\
m_\mu \\
M_0[V,A] \\
m_\mu
\end{array}
\]

c) \[
\begin{array}{c}
H^- \\
H^+ \\
X^0
\end{array}
\]

d) \[
\begin{array}{c}
X^- \\
X^+ \\
X^0
\end{array}
\]

Neutral boson exchange: a) scalar or pseudoscalar and b) vector or axialvector, flavor changing or not.

New charged bosons: c) scalars or pseudoscalars, d) vector or axialvector.

Neutral boson of mass $M_0$ coupling to muons with strength $f$ from diagrams (a) and (b) [Lautrup, Peterman, de Rafael '72; Leveille '78]:

\[
\Delta a_{\mu}^{NP} = \frac{f^2 m_\mu^2}{4\pi^2 M_0^2 L}, \quad L = \frac{1}{2} \int_0^1 dx \frac{Q(x)}{(1 - x) (1 - \lambda^2 x) + (\epsilon \lambda)^2 x}
\]

where $Q(x)$ is a polynomial in $x$ which depends on the type of coupling:

- Scalar : $Q_S = x^2 (1 + \epsilon - x)$
- Pseudoscalar : $Q_P = x^2 (1 - \epsilon - x)$
- Vector : $Q_V = 2x (1 - x) (x - 2 (1 - \epsilon)) + \lambda^2 (1 - \epsilon)^2 Q_S$
- Axialvector : $Q_A = 2x (1 - x) (x - 2 (1 + \epsilon)) + \lambda^2 (1 + \epsilon)^2 Q_P$

with $\epsilon = M/m_\mu$ and $\lambda = m_\mu/M_0$. For $m_\mu, M \ll M_0$ one obtains, for instance,

\[
L_S = \frac{M}{m_\mu} \left( \ln \frac{M_0}{M} - \frac{3}{4} \right) + \frac{1}{6} M = m_\mu \ln \frac{M_0}{m_\mu} - \frac{7}{12}
\]

In accordance with Minimal Flavor Violation requirement it is more realistic to assume a flavor conserving neutral current $M = m_\mu$ (second form). Expressions for all $L$ for $m_\mu, M \ll M_0$ and $m_\mu \ll M \sim M_0$ (important for SUSY), see Jegerlehner, AN '09.
Generic 1-loop New Physics contributions to \( g - 2 \) (continued)

(a) \( m_\mu = M \ll M_0 \)

Single particle one–loop induced NP effects from neutral bosons for \( f^2/(4\pi^2) = 0.01 \).

(Note, a typical EW SM coupling would be \( e^2/(4\pi^2 \cos^2 \Theta_W) = 0.003 \)). In panel (b),

the large chiral combinations \( L_S - L_P \) and \( L_V - L_A \) are rescaled by the muon Yukawa
coupling \( m_\mu/v \) in order to compensate for the huge pre-factor \( M/m_\mu \).

New physics contributions due to charged S,P,V and A modes from the diagrams (c) and (d):

\[
\Delta a_{\mu}^{\text{NP}} = \frac{f^2}{4\pi^2} \frac{m_\mu^2}{M_0^2} L, \quad L = \frac{1}{2} \int_0^1 dx \frac{Q(x)}{(\epsilon \lambda)^2 (1-x) (1-\epsilon^{-2}x) + x}
\]

where again \( Q(x) \) is a polynomial in \( x \) which depends on the type of coupling:

Scalar \quad : \quad Q_S = -x (1-x) (x + \epsilon) 

Pseudoscalar \quad : \quad Q_P = -x (1-x) (x - \epsilon) 

Vector \quad : \quad Q_V = -2x^2 (1+x-2\epsilon) + \lambda^2 (1-\epsilon)^2 Q_S 

Axialvector \quad : \quad Q_A = -2x^2 (1+x+2\epsilon) + \lambda^2 (1+\epsilon)^2 Q_P 

Expressions for \( L \) in the limits given earlier can be found in Jegerlehner, AN '09.
**$a_\mu$: Supersymmetry**

Supersymmetry for large $\tan \beta, \mu > 0$:

$$a_\mu^{\text{SUSY}} \approx 123 \times 10^{-11} \left( \frac{100 \text{ GeV}}{M_{\text{SUSY}}} \right)^2 \tan \beta$$

(Czarnecki, Marciano, 2001)

Explains $\Delta a_\mu = 290 \times 10^{-11}$ if $M_{\text{SUSY}} \approx (93 - 414) \text{ GeV}$ \ ($2 < \tan \beta < 40$).

In some regions of parameter space, large 2-loop contributions (2HDM):

Barr–Zee diagram (b) yields enhanced contribution, which can exceed the 1–loop result. **Enhancement factor** $m_b^2/m_\mu^2$ **compensates suppression by** $\alpha/\pi$ (($\alpha/\pi \times (m_b^2/m_\mu^2) \sim 4 > 1$).

Constraint on large $\tan \beta$ SUSY contributions as function of $M_{\text{SUSY}}$.

Allowed values of MSSM contributions to $a_\mu$ as function of mass of Lightest Observable SUSY Particle $M_{\text{LOSP}}$ (MSSM parameter scan with $\tan \beta = 50$. Courtesy D. Stöckinger, '09.)
Pre-LHC parameter space in constrained MSSM (CMSSM) (Courtesy K. Olive, '09):

\[(m_0, m_{1/2}) \text{ plane for } \mu > 0 \text{ for } \tan \beta = 10 \text{ (left panel) and } \tan \beta = 40 \text{ (right panel) in CMSSM. } a_\mu \text{ favored region at } 2\sigma \text{ level } [(290 \pm 180) \times 10^{-11}] \text{ in pink. Region excluded by } b \rightarrow s\gamma \text{ in green. Allowed region by cosmological neutral dark matter constraint is black parabolic shaped region. Disallowed region where } m_{\tilde{\tau}_1} < m_\chi \text{ in brown.} \]
$a_\mu$ and Supersymmetry after first LHC run

- **Global SUSY fits** (EW precision tests, flavor physics ($a_\mu$, $b$-decays), $m_h$, Dark Matter relic density, Dark Matter searches, LHC searches) by several groups (with year of last publication): The BayesFITS group (2012); Fittino (2012); The MasterCode Project (2015); SuperBayeS (2011).

- Note: muon $g - 2$ and SUSY searches at LHC only lead to tension in CMSSM or NUHM1 / NUHM2 (non-universal contributions to Higgs masses).

- LHC so far only sensitive to strongly interacting supersymmetric particles, like squarks and gluinos (ruled out below about 1 TeV).

- In general supersymmetric models (e.g. pMSSM10 = phenomenological MSSM with 10 soft SUSY-breaking parameters) with light neutralinos, charginos and sleptons, one can still explain muon $g - 2$ discrepancy and evade bounds from LHC.
Parameters in pMSSM10 (specified at low renormalization scale close to EWSB scale: mean scalar top mass $M_{SUSY} \equiv \sqrt{m_{\tilde{t}_1} m_{\tilde{t}_2}}$):

- 3 gaugino masses: $M_{1,2,3}$
- 2 squark masses: $m_{\tilde{q}_1} = m_{\tilde{q}_2} \neq m_{\tilde{q}_3}$
- 1 slepton mass: $m_{\tilde{\ell}}$
- 1 trilinear coupling: $A$
- Higgs mixing parameter: $\mu$
- Pseudoscalar Higgs mass: $M_A$
- Ratio of vevs: $\tan \beta$

<table>
<thead>
<tr>
<th>Constraint</th>
<th>d.o.f.</th>
<th>best fit</th>
<th>CMSSM</th>
<th>NUHM1</th>
<th>NUHM2</th>
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<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Jets+$E_T$</td>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<tr>
<td>$M_h$</td>
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<td>2.0</td>
<td>0.0</td>
<td>0.5</td>
</tr>
<tr>
<td>$M_W$</td>
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<td>0.1</td>
<td>0.5</td>
<td>0.1</td>
</tr>
<tr>
<td>$B_{s,d} \to \mu^+\mu^-$</td>
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<td>0.3</td>
<td>0.4</td>
</tr>
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<td>BR($b \to s \gamma$)</td>
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<td>0.5</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
<td>BR($B_u \to \tau\nu_\tau$)</td>
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<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Other $B$ physics</td>
<td>5</td>
<td>3.3</td>
<td>3.2</td>
<td>3.3</td>
<td>3.3</td>
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<td>$\Omega \chi_1^0 h^2$</td>
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<td>0.1</td>
<td>0.0</td>
<td>0.0</td>
<td>0.0</td>
</tr>
<tr>
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<td>0.0</td>
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<tr>
<td>$A/H \to \tau^+\tau^-$</td>
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<td>0.0</td>
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<tr>
<td>$(g - 2)_\mu$</td>
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<table>
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</tr>
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<tbody>
<tr>
<td>$\chi^2$/d.o.f.</td>
<td>20.5/18</td>
<td>22.2/18</td>
<td>22.0/18</td>
<td>22.3/18</td>
<td>22.2/18</td>
<td>32.8/24</td>
<td>31.1/23</td>
<td>30.3/22</td>
</tr>
<tr>
<td>$\chi^2$ probability</td>
<td>0.31</td>
<td>0.22</td>
<td>0.23</td>
<td>0.22</td>
<td>0.22</td>
<td>0.11</td>
<td>0.12</td>
<td>0.11</td>
</tr>
</tbody>
</table>

$\chi^2$ breakdowns at pMSSM10 best-fit and low-$m_{\tilde{t}_1}$, low-$m_{\tilde{q}}$ and low-$m_{\tilde{g}}$ points and in CMSSM, NUHM1, NUHM2. For each set of constraints, the $\chi^2$ contribution and the number of non-zero contributions is provided. Nuisance parameters: $m_t$, $\alpha_s(M_Z)$, $M_Z$. 

Profile likelihoods for the SUSY contribution to $(g - 2)\mu$

Left panel shows $\Delta\chi^2$ contributions from $(g - 2)\mu$ to global likelihood functions of fits to CMSSM (blue dotted line), NUHM1 (blue dashed line), NUHM2 (blue solid line) and pMSSM10 (black solid line), as well as the assumed experimental likelihood function (solid red line). Right panel displays global $\chi^2$ function calculated without (dashed line) and with (solid line) the contribution of the electroweakly-interacting sparticle searches implemented via LHC8_{EWK}.
**$a_e, a_\mu$: Dark photon**

In some dark matter scenarios, there is a relatively light, but massive "dark photon" $A'_\mu$ that couples to the SM through mixing with the photon:

$$L_{\text{mix}} = \frac{2}{\varepsilon} F_{\mu\nu} F'_{\mu\nu}$$

$\Rightarrow$ $A'_\mu$ couples to ordinary charged particles with strength $\varepsilon \cdot e$.

$\Rightarrow$ additional contribution of dark photon with mass $m_{\gamma'}$ to the $g-2$ of a lepton (electron, muon) (Pospelov '09):

$$a_{\ell}^{\text{dark photon}} = \frac{\alpha}{2\pi} \varepsilon^2 \int_0^1 dx \frac{2x(1-x)^2}{(1-x)^2 + \frac{m_{\gamma'}^2}{m_{\ell}^2} x}$$

$$= \frac{\alpha}{2\pi} \varepsilon^2 \times \begin{cases} 1 & \text{for } m_{\ell} \gg m_{\gamma'} \\ \frac{2m_{\ell}^2}{3m_{\gamma'}^2} & \text{for } m_{\ell} \ll m_{\gamma'} \end{cases}$$

For values $\varepsilon \sim (1 - 2) \times 10^{-3}$ and $m_{\gamma'} \sim (10 - 100) \text{ MeV}$, the dark photon could explain the discrepancy $\Delta a_\mu = 290 \times 10^{-11}$.

Various searches for the dark photon have been performed, are under way or are planned at BABAR, Jefferson Lab, KLOE, MAMI and other experiments. For a recent overview, see: *Dark Sectors and New, Light, Weakly-Coupled Particles* (Snowmass 2013), Essig et al., arXiv:1311.0029 [hep-ph].
Status of dark photon searches

Essentially all of the parameter space in the $(m_{\gamma'}, \varepsilon)$-plane to explain the muon $g - 2$ discrepancy has now been ruled out.

From: F. Curciarello, FCCP15, Capri, September 2015
Conclusions

• Over many decades, the (anomalous) magnetic moments of the electron and the muon have played a crucial role in atomic and elementary particle physics.

• Experiment and Theory were thereby often going hand-in-hand, pushing each other to the limits.

• From $a_e, a_\mu$ we gained important insights into the structure of the fundamental interactions (quantum field theory).

• $a_e$: Test of QED, precise determination of fine-structure constant $\alpha$.
  $a_\mu$: Test of Standard Model, potential window to New Physics.
Outlook

- Current situation:
  \[ a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = (278 \pm 88) \times 10^{-11} \quad [3.1 \sigma] \]
  Sign of New Physics ? Hadronic effects ? Does \( g - 2 \) experiment measure something different from what is calculated in theory?

- Two new planned \( g - 2 \) experiments at Fermilab and J-PARC with goal of \( \delta a_\mu^{\text{exp}} = 16 \times 10^{-11} \) (factor 4 improvement)

- Theory needs to match this precision !

- Hadronic vacuum polarization
  Ongoing and planned experiments on \( \sigma(e^+e^- \rightarrow \text{hadrons}) \) with a goal of \( \delta a_\mu^{\text{HVP}} = (20 - 25) \times 10^{-11} \) (factor 2 improvement)

- Hadronic light-by-light scattering
  \[
  a_\mu^{\text{HLbL}} = (105 \pm 26) \times 10^{-11} \quad \text{(Prades, de Rafael, Vainshtein '09)}
  \]
  \[
  a_\mu^{\text{HLbL}} = (116 \pm 40) \times 10^{-11} \quad \text{(AN '09; Jegerlehner, AN '09)}
  \]
  Error estimates are mostly guesses ! Need a much better understanding of the complicated hadronic dynamics to get reliable error estimate of \( \pm 20 \times 10^{-11} \).

- Better theoretical models needed; more constraints from theory (ChPT, pQCD, OPE); close collaboration of theory and experiment to measure the relevant decays, form factors and cross-sections of hadrons with photons.
- Promising new data-driven approach using dispersion relations for \( \pi^0, \eta, \eta' \) and \( \pi\pi \). Still needed: data for scattering of off-shell photons.
- Future: Lattice QCD.
And finally:

**g-2 measuring the muon**

In the 1950s, the muon was still a complete enigma. Physicists could not yet say with certainty whether it was simply a much heavier electron or whether it belonged to another species of particle. g-2 was set up to test quantum electrodynamics, which predicts, among other things, an anomalously high value for the muon’s magnetic moment \(g\), hence the name of the experiment.

The second g-2 experiment started in 1966 under the leadership of Francis Farley and it achieved a precision of 0.4% with respect to the theoretical value. By 1962, this precision had been whittled down to just 0.4%. This was a great success since it validated the theory of quantum electrodynamics, which predicted that the muon turned out to be a heavy electron.

“g-2 is not an experiment: it is a way of life.” John Adams

A third experiment, with a new technical approach, was launched in 1969, under the leadership of Emilio Picasso. The final results were published in 1979 and confirmed the theory to a precision of 0.0007%. They also allowed observation of a phenomenon contributing to the muon’s magnetic moment, the presence of ‘virtual hadrons’. After 1984, the United States took up the mantle of investigating the muon’s anomalous magnetic moment, applying the findings towards an accurate test of QED.

This statement also applies to many theorists working on the \(g-2\) !
Backup slides
Anomalous magnetic moment in quantum field theory

Quantized spin 1/2 particle interacting with external, classical electromagnetic field

4 form factors in vertex function

(momentum transfer $k = p' - p$, not assuming parity or charge conjugation invariance)

\[
\equiv i\langle p', s' | j^\mu(0) | p, s \rangle
=\begin{pmatrix}
-\begin{pmatrix}
\gamma^\mu F_1(k^2) + \frac{i\sigma^{\mu\nu}k_\nu}{2m} F_2(k^2) \\
\text{Dirac} \\
\text{Pauli}
\end{pmatrix}
+\gamma^5 \frac{\sigma^{\mu\nu}k_\nu}{2m} F_3(k^2) + \gamma^5 (k^2\gamma^\mu - k^\mu k^\nu) F_4(k^2)
\end{pmatrix} u(p, s)
\]

$k = \gamma^\mu k_\mu$. Real form factors for spacelike $k^2 \leq 0$. Non-relativistic, static limit:

\[
F_1(0) = 1 \quad \text{(renormalization of charge $e$)}
\]
\[
\mu = \frac{e}{2m} (F_1(0) + F_2(0)) \quad \text{(magnetic moment)}
\]
\[
a = F_2(0) \quad \text{(anomalous magnetic moment)}
\]
\[
d = -\frac{e}{2m} F_3(0) \quad \text{(electric dipole moment, violates P and CP)}
\]
\[
F_4(0) = \text{anapole moment (violates P)}
\]
HLbL in muon $g-2$: summary of selected results

Some results for the various contributions to $a_{\mu}^{HLbL} \times 10^{11}$:

<table>
<thead>
<tr>
<th>Contribution</th>
<th>BPP</th>
<th>HKS, HK</th>
<th>KN</th>
<th>MV</th>
<th>BP, MdRR</th>
<th>PdRV</th>
<th>N, JN</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^0, \eta, \eta'$</td>
<td>85±13</td>
<td>82.7±6.4</td>
<td>83±12</td>
<td>114±10</td>
<td>—</td>
<td>114±13</td>
<td>99±16</td>
</tr>
<tr>
<td>axial vectors</td>
<td>2.5±1.0</td>
<td>1.7±1.7</td>
<td>—</td>
<td>22±5</td>
<td>—</td>
<td>15±10</td>
<td>22±5</td>
</tr>
<tr>
<td>scalars</td>
<td>−6.8±2.0</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−7±7</td>
<td>−7±2</td>
</tr>
<tr>
<td>$\pi, K$ loops</td>
<td>−19±13</td>
<td>−4.5±8.1</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>−19±19</td>
<td>−19±13</td>
</tr>
<tr>
<td>$\pi, K$ loops + subl. $N_C$</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>0±10</td>
<td>−</td>
<td>−</td>
<td>−</td>
</tr>
<tr>
<td>quark loops</td>
<td>21±3</td>
<td>9.7±11.1</td>
<td>−</td>
<td>−</td>
<td>−</td>
<td>2.3 (c-quark)</td>
<td>21±3</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>83±32</td>
<td>89.6±15.4</td>
<td>80±40</td>
<td>136±25</td>
<td>110±40</td>
<td>105±26</td>
<td>116±39</td>
</tr>
</tbody>
</table>

BPP = Bijnens, Pallante, Prades '95, '96, '02; HKS = Hayakawa, Kinoshita, Sanda '95, '96; HK = Hayakawa, Kinoshita '98, '02; KN = Knecht, AN '02; MV = Melnikov, Vainshtein '04; BP = Bijnens, Prades '07; MdRR = Miller, de Rafael, Roberts '07; PdRV = Prades, de Rafael, Vainshtein '09; N = AN '09, JN = Jegerlehner, AN '09

- **Pseudoscalar-exchanges dominate numerically.** Other contributions not negligible. Cancellation between $\pi, K$-loops and quark loops!

- **PdRV:** Analyzed results obtained by different groups with various models and suggested new estimates for some contributions (shifted central values, enlarged errors). Do not consider dressed light quark loops as separate contribution! Assume it is already taken into account by using short-distance constraint of MV '04 on pseudoscalar-pole contribution. Added all errors in quadrature!

- **N, JN:** New evaluation of pseudoscalar exchange contribution imposing new short-distance constraint on off-shell form factors. Took over most values from BPP, except axial vectors from MV. Added all errors linearly.
HLbL: recent developments

• Most calculations for neutral pion and all light pseudoscalars agree at level of 15%, but some are quite different:
  \[ a_{\mu}^{\text{HLbL};\pi^0} = (50 - 80) \times 10^{-11} \]
  \[ a_{\mu}^{\text{HLbL};\text{PS}} = (59 - 114) \times 10^{-11} \]

• New estimates for axial vectors (Pauk, Vanderhaeghen ’14; Jegerlehner ’14):
  \[ a_{\mu}^{\text{HLbL};\text{axial}} = (6 - 8) \times 10^{-11} \]
  Substantially smaller than in MV ’04!

• First estimate for tensor mesons (Pauk, Vanderhaeghen ’14):
  \[ a_{\mu}^{\text{HLbL};\text{tensor}} = 1 \times 10^{-11} \]

• Open problem: Dressed pion-loop
  Potentially important effect from pion polarizability and \( a_1 \) resonance (Engel, Patel, Ramsey-Musolf ’12; Engel ’13; Engel, Ramsey-Musolf ’13):
  \[ a_{\mu}^{\text{HLbL};\pi-\text{loop}} = -(11 - 71) \times 10^{-11} \]
  Maybe large negative contribution, in contrast to BPP ’96, HKS ’96.

• Open problem: Dressed quark-loop
  Dyson-Schwinger equation approach (Fischer, Goecke, Williams ’11, ’13):
  \[ a_{\mu}^{\text{HLbL};\text{quark-loop}} = 107 \times 10^{-11} \]
  Large contribution, no damping seen, in contrast to BPP ’96, HKS ’96.
Supersymmetry (pMSSM10) after first LHC run


Particle spectrum and dominant decay branching ratios at best-fit pMSSM10 point

Note near-degeneracies between $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^\pm_1$, between sleptons, between $\tilde{\chi}^0_3$, $\tilde{\chi}^0_4$, $\tilde{\chi}^\pm_2$, between $\tilde{q}_L$, $\tilde{q}_R$, between the heavy Higgs bosons, and between stops and bottoms. The overall sparticle mass scales, in particular of colored sparticles, are poorly determined.

Summary of mass ranges predicted in the pMSSM10

Light (darker) peach shaded bars indicate 95% (68%) CL intervals, blue horizontal lines mark values of masses at best-fit point.
Supersymmetry (pMSSM10) after first LHC run (continued)


Particle spectra and dominant decay branching ratios at the four benchmark points

Upper left panel: the low-$m_{\tilde{t}_1}$ pMSSM10 point, where stops and bottoms are relatively light. Upper right panel: similarly for low-$m_{\tilde{q}}$ benchmark point, where all the squarks are relatively light. Lower left panel: similarly for low-$m_{\tilde{g}}$ benchmark point. Lower right panel: similarly for the point where all squarks and the gluino masses are $< 2$ TeV. Note in each case the near-degeneracies between $\tilde{\chi}^0_1$, $\tilde{\chi}^0_2$, $\tilde{\chi}^{\pm}_1$, between the sleptons, between $\tilde{\chi}^0_3$, $\tilde{\chi}^0_4$, $\tilde{\chi}^{\pm}_2$, between $\tilde{q}_L$, $\tilde{q}_R$, and between the heavy Higgs bosons.