Effects of Minkowskian evolution on CMB bispectrum

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Outline

- Introduction.
- Novel picture of cosmological perturbations evolution.
- Non-Gaussianities.
- Imprint on CMB.
- Conclusions.
Large scale structures and primordial seeds

Universe is homogeneous and isotropic at large scales, but...

Structures:
- Clusters of galaxies, $D \sim 1 - 5$ Mpc
- Galaxies, $D \sim 5 - 250$ kpc
- Smaller structures

Seeds for future structures.

$\delta \rho / \rho \sim \Phi \sim 10^{-5}$.

During evolution perturbations grow

$\delta \rho / \rho \sim 1$.

Enter non-linear regime $\implies$ gravitational collapse occurs.
Planck data has come...

- Nearly scale invariant power spectrum,

\[ \langle \Phi(k)\Phi(k') \rangle = \frac{1}{4\pi k^3} P_\delta(k)\delta(k + k') \]

- Slight negative tilt Ade et al.'13

\[ \frac{\partial P_\delta(k)}{\partial \ln k} = n_s - 1 \quad n_s = 0.9603 \pm 0.0073 \text{ (68% CL)} \]

- Also Gaussian with a high accuracy. In particular, the bispectrum

\[ \langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle \]

is consistent with zero.

Inflation: period of rapid quasi-de Sitter expansion Starobinski’79 Guth’81 Sato’81 Linde’82 and ’83 Albrecht and Steinhardt’82
Enhanced vacuum fluctuations of inflaton as the source of primordial scalar perturbations with nearly flat spectrum Mukhanov and Chibisov’81.

In single-field slow roll inflation with the canonical kinetic term, also Gaussian with a high accuracy Maldacena’03.

However, multi-field inflation (inflaton + curvaton)

$$\Phi(x) = \Phi_G(x) + f_{NL}(\Phi_G^2(x) - \langle \Phi_G^2(x) \rangle)$$

$f_{NL} \gg 1$.

Local non-Gaussianity—squeezed bispectrum, amplified as $k_i \to 0$.

$$\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle \sim f_{NL} \cdot P^2_\Phi \cdot \frac{\sum_i k_i^3}{\prod_i k_i^3} \delta \left( \sum_i k_i \right)$$

In single-field case in the limit $k_i \to 0$  

$$f_{NL} \sim |n_s - 1| \sim 0.01.$$  

$$f_{NL} = 2.7 \pm 5.8 \text{ at 68% CL Ade et al.’13}$$
Huge number of models. Any selection principle?

Non-Gaussianities are very sensitive to the state of cosmological perturbations at times they are generated (Babich et al.’04)

Cosmological perturbations start from vacuum initial conditions, leave the horizon, remain unchanged until hot era.

Super-h regime: \( k_i \rightarrow 0 \)

On-h regime: \( k_1 \sim k_2 \sim k_3 \)

Excited initial conditions:

\( k_i + k_j = k_s \) Holman and Tolley’07

Many models are constrained by Planck \( \Rightarrow \) We are blocked!
What if cosmological perturbations followed a drastically different evolution?—Good chance to obtain something new...
Cosmological perturbations start from vacuum initial conditions.

Get frozen out at some point, and acquire nearly flat power spectrum.

Long intermediate stage, when space-time is nearly Minkowskian and perturbations oscillate. Alternative to inflation!

\[ |\eta_*| \gg k^{-1} \]

Leave the horizon and remain unchanged until hot era.
Nearly Minkowski metric?

- Slow contraction $P \gg \rho$. Ekpyrosis (Khoury et al.’01)
  - Conformal Universe (Hinterbichler and Khoury’11 Rubakov’09)
  - Nearly static Universe $\rightarrow$ bounce $\rightarrow$ hot Big Bang
- Or slow expansion: Galilean Genesis Creminelli et al.’10.
  - Nearly static at early times $\rightarrow$ rapid expansion $\rightarrow$ hot Big Bang.

Just Minkowski metric is not enough... How to freeze out modes?

Usually $\theta \leftrightarrow g^{\mu \nu}$. But... freezing out $\neq$ leaving the horizon.

In Conformal Universe: due to interaction with some other field $\theta \leftrightarrow \rho$.

Interaction becomes irrelevant, but modes can be still sub-horizon.
Evolution during intermediate times: $\eta_* < \eta < 0$

Initial conditions \[ \theta(k, \eta) \big|_{\eta_*} = \theta(k, \eta_*) \quad \partial_\eta \theta(k, \eta) \big|_{\eta_*} = 0. \]

Assume massless evolution, non-affected by other fields.

Equation \[ \theta'' - \partial_i^2 \theta = 0 \]

Solution \[ \theta(k, \eta) = \theta(k, \eta_*) \cos k(\eta_* - \eta) \ (\eta_* < \eta < 0). \]

\[ \theta(k, 0) = \theta(k, \eta_*) \cos k\eta_* \rightarrow \Phi(k) \quad \theta(k, 0) \sim \Phi(k). \]

Conversion mechanisms: curvaton-type Dimopoulos et al.’03, modulated decay Dvali et al.’03 Kofman’03.
Bispectrum

\[
\langle \Phi(k_1)\Phi(k_2)\Phi(k_3) \rangle = \text{const} \langle \theta(k_1, \eta_*)\theta(k_2, \eta_*)\theta(k_3, \eta_*) \rangle \times
\]

\[
\times \cos(k_1 \eta_*) \cos(k_2 \eta_*) \cos(k_3 \eta_*)
\]

\[
\frac{1}{4} \left[ \cos(k_1 + k_2 + k_3)\eta_* + \cos(k_1 + k_2 - k_3)\eta_* + \ldots \right]
\]

\[
\langle \theta(k_1, \eta_*)\theta(k_2, \eta_*)\theta(k_3, \eta_*) \rangle = A(k_1, k_2, k_3)\delta \left( \sum_i k_i \right)
\]

Keep \( A(k_1, k_2, k_3) \) as a generic slow varying function.

\[
|\eta_*| \gg k^{-1} \implies \text{cosines rapidly oscillate} \implies k_i + k_j \approx k_s
\]

Cosines act independently of the function \( A(k_1, k_2, k_3) \).
Moving towards CMB

\[ \langle \theta(\mathbf{k}_1)\theta(\mathbf{k}_2)\theta(\mathbf{k}_3) \rangle \rightarrow \langle \Phi(\mathbf{k}_1)\Phi(\mathbf{k}_2)\Phi(\mathbf{k}_3) \rangle \rightarrow \langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle. \]

\[ a_{l m} = \int d\Omega_n Y^*_{l m}(n) \delta T(n). \]

\[ a_{l m} = 4\pi i^l \int \frac{d\mathbf{k}}{(2\pi)^{3/2}} \Delta_l(k\eta_0)\Phi(\mathbf{k}) Y^*_{l m}(n_k). \]

Transfer function \( \Delta_l(k\eta_0) = g_l(k)j_l(k\eta_0) \).

The major peak is at \( k \sim l/\eta_0 \)

\[ \eta_0 \sim H_0^{-1} \sim 14 \text{ Gpc} \quad (a_0 = 1) \]
Since the statistical anisotropy,

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle \sim \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix} b_{l_1 l_2 l_3}$$

In the regime $$|\eta_*| \gtrsim l_{\text{min}} \eta_0$$.

$$\cos(k_1 \eta_*) \cos(k_2 \eta_*) \cos(k_3 \eta_*) \implies b_{l_1 l_2 l_3} \propto \frac{\eta_0^2}{\eta_*^2}$$

Bispectrum vanishes as $$|\eta_*| \to \infty$$. Of the observable size, if $$|\eta_*| \sim \eta_0$$.

**Naive expectations:** since $$k_i + k_j = k_s$$ and $$k_i \sim l_i$$, then $$l_i + l_j = l_s$$.

Flattened triangle limit.

**NB** In inflation: from excited initial conditions Holman and Tolley’07

However…
Expectations

Unexpectedly...

\[ \Delta l = l_1 + l_2 - l_3 \]

\( b_{l_1 l_2 l_3} \)

Reality

\( l_1 = 50 \)

\( l_2 = 600 \)
The bispectrum is distinct from inflationary bispectra.

Oscillations are traced back to oscillations of transfer functions

\[ \Delta_l \equiv l_1 + l_2 - l_3 \]

\[ \Delta_l(k \eta_0) \sim j_l(k \eta_0) \text{ (period } \sim l^{1/3} \sim 10) \]
Prospects for Planck: pessimism

Specify to the particular model: conformal rolling scenario.

Of the observable size in the region:

\[ \eta_0 \lesssim |\eta_*| \lesssim 10 \cdot \eta_0. \]

On the other hand, calculations can be trusted in the region

\[ |\eta_*| \gtrsim l_{\text{min}} \eta_0 \sim 10 - 100 \cdot \eta_0. \]

Bispectrum \( A(k_1, k_2, k_3) \) is large enough.
Prospects for Planck: optimism

What about the region $\eta_0 \lesssim |\eta_*| \lesssim l_{\text{min}} \cdot \eta_0$?  

Work in Progress!

Indications: shapes remain the same.

Suppression

$$\frac{\eta_0^2}{\eta_*^2} \rightarrow \frac{\eta_0}{|\eta_*| \cdot l_{\text{min}}}$$

For example,

$$l_{\text{min}} = 100 \implies |\eta_*| \sim \eta_0 .$$

$$l_{\text{min}} = 10 \implies |\eta_*| \sim 10 \cdot \eta_0 .$$
Conclusions

- Non-trivial evolution of perturbations in the nearly Minkowski background leads to distinct shapes of bispectra.

- Bispectrum can be of the observable size provided that the duration $|\eta_*| \sim H_0^{-1}$ (Work in progress)
MERCI ! BEDANKT !