Dark matter distribution around massive black holes and in phase-space

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Outline

Introduction: detecting dark matter

The DM distribution around a supermassive BH

DM annihilation in phase-space

Phase space distribution: Eddington’s inversion

Conclusions


FF & Daniel Hunter, JCAP 1309, 005 (2013) [arXiv:1306.6586]
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The case for dark matter

Most economical explanation of:

- The rate of expansion of the universe.
- The formation of large scale structure.
- The dynamics of galaxies, clusters, . . .

Expected in natural extensions of the SM.
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Expected in natural extensions of the SM.
An example: WIMPs

Similar to a heavy neutrino, $m_\chi \approx 100$ GeV, weak-scale interactions produce observed abundance from thermal decoupling:

$$\Rightarrow <\sigma v> \approx 3 \times 10^{-26} \text{ cm}^3 \text{ s}^{-1}$$

The same interactions make it potentially detectable:

- $\chi\chi \rightarrow \gamma\gamma, \pi^0, e^\pm, \ldots$
- $\chi N \rightarrow \chi N$

Other examples include axions, MeV particles, \ldots
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Indirect detection

\[ Flux = \frac{\langle \sigma v \rangle}{4\pi m_{dm}^2} \frac{dN_\gamma}{dE_\gamma} \left\{ \begin{array}{c} \text{Number of SM particles} \\ \text{Amount of DM} \end{array} \right\} \times \int_0^\infty \rho^2(r) dl \]

- **Astrophysical factor** suggests looking at GC, dwarf spheroidals, . . .
- Photons and neutrinos point back to the source, while charged particles diffuse.
The distribution of DM: simulations

1 billion 4,100 $M_\odot$ particles. 0.5 kpc in the host halo.
The distribution of DM: observations

Jeans’ equation shows that $M/L \sim 1000$. Clean systems.
The central supermassive black hole

- Will focus on the super-massive BH at the center of the Galaxy.
- Similar effects will occur in the cores of AGNs, or in IMBHs.
The central supermassive black hole

We will assume that a black hole of mass $4 \times 10^6 M_\odot$ grows adiabatically over $\sim 10^{10}$ yr.
The central supermassive black hole

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Is the growth adiabatic?

- **Growth time**
- **Dynamical time**

\[
\frac{r_h}{\sigma} \leq \frac{m}{\dot{m}_{\text{Edd}}}
\]

\[
r_h \approx \frac{Gm}{\sigma^2} \quad \Rightarrow \quad t_{\text{dyn}} \approx 10^4 \text{yr} \leq t_{\text{Salpeter}} \approx 5 \times 10^7 \text{yr}
\]

Caveats: Hierarchical mergers, initial BH seed off-center, kinetic heating of DM by stars, ...
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Growing a BH: Newtonian analysis

We are interested in the DM density:

\[\rho = \int f(E, L) d^3v\]

\[= 4 \int dE \int LdL \int dL_z \frac{f(E, L)}{r^4 \left| v_r \right| \left| v^\theta \right| \sin \theta}\]

\[= 4\pi \int dE \int LdL \frac{f(E, L)}{r^2 \left| v_r \right|}\]

The limits of integration are set by the requirements:

- \(\left| v_r \right| = (2E - 2\Phi - L^2/r^2)^{1/2} \text{ real} \Rightarrow 0 \leq L \leq [2r^2(E - \Phi)]^{1/2}\).
- DM particle is bound to the halo \(\Rightarrow \Phi(r) \leq E \leq 0\).

Take into account particles trapped inside the event horizon by modifying boundary conditions in an *ad hoc* manner: \(L \geq 2cR_S\).
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Take into account particles trapped inside the event horizon by modifying boundary conditions in an \textit{ad hoc} manner: \( L \geq 2cR_S. \)
Growing a BH: Newtonian analysis

Each particle in an initial DM distribution $f(E, L)$, will react to the change in $\Phi$ caused by the growth of the BH by altering its $E$, $L$ and $L_z$. However, the adiabatic invariants remain fixed:

$$I_r(E, L) \equiv \oint v_r dr = \oint dr \sqrt{2E - 2\Phi - L^2/r^2} ,$$
$$I_\theta(L, L_z) \equiv \oint v_\theta d\theta = \oint d\theta \sqrt{L^2 - L_z^2 \sin^{-2} \theta} = 2\pi (L - L_z) ,$$
$$I_\phi(L_z) \equiv \oint v_\phi d\phi = \oint L_z d\phi = 2\pi L_z . \quad (1)$$

The shape of the distribution function is also invariant, $f(E, L) = f'(E'(E, L), L)$. 
Growing a BH: Newtonian analysis

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\] (1)

The shape of the distribution function is also invariant, \( f(E, L) = f'(E'(E, L), L) \).
Newtonian BH

For a Newtonian point mass,

\[ I_r(E, L) = 2\pi \left( -L + \frac{Gm}{\sqrt{-2E}} \right) \]

And we can find the final DM density in the form:

\[ \rho(r) = \frac{4\pi}{r^2} \int_{-Gm/r}^{0} dE \int_{0}^{L_{\text{max}}} LdL \frac{f'(E'(E, L), L)}{\sqrt{2E + 2Gm/r - L^2/r^2}} \]

Young 80, Quinlan et al. 95, Gondolo & Silk 95
Growing a BH: Relativistic analysis

1. Generalize the definition of density:

\[ J^\mu(x) \equiv \int f^{(4)}(p)p^\mu_\mu \sqrt{-g} \, d^4p, \]

2. Use relativistic expressions to write it in terms of the invariants of motion (energy, angular momentum, . . .).

3. Use relativistic expressions for the actions.

For Kerr:

\[ \mathcal{E} \equiv -u_0 = -g_{00}u^0 - g_{0\phi}u^\phi, \]
\[ L_z \equiv u_\phi = g_{0\phi}u^0 + g_{\phi\phi}u^\phi, \]
\[ C \equiv \Sigma^4(u^\theta)^2 + \sin^{-2}\theta L_z^2 + a^2 \cos^2\theta(1 - \mathcal{E}^2), \]
\[ g_{\mu\nu}p^\mu p^\nu = -\mu^2. \]

And need to calculate the jacobian

\[ d^4p = |J|^{-1} d\mathcal{E} dCdL_z d\mu \]
Growing a BH: Relativistic analysis

1. Generalize the definition of density:

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And need to calculate the jacobian

\[ d^4 p = |J|^{-1} dE dC dL_z d\mu \]
Example: Schwarzschild BH

The positivity of the radial action determines the boundary conditions, \textit{including the effects of the horizon.}
Example: Schwarzschild BH

For a constant phase-space distribution:

\[
\rho = 10^8 \text{ GeV/cm}^3
\]

Gondolo & Silk
Example: Schwarzschild BH

For a more realistic, cuspy DM distribution:

\[
\log_{10}(r/R_S)
\]

\[
\log_{10}(\rho \text{ [GeV/cm}^3])
\]

Relativistic
Non-relativistic
DM annihilation
Initial Hernquist profile
Consequences

The gravitational potential is still dominated by the BH:

\[ \log_{10} \left( \frac{r}{R_S} \right) \]

\[ \log_{10} \left( \frac{m(r)}{M_\odot} \right) \]

- no annihilation
- with annihilation

[Graph showing the gravitational potential as a function of \( \log_{10} \left( \frac{r}{R_S} \right) \) and \( \log_{10} \left( \frac{m(r)}{M_\odot} \right) \).]
Consequences

No big changes for DM annihilation, but precession rates could be observable.

![Graph showing precession rates and semi-major axes](image-url)
Indirect detection

\[
\text{Flux} = \frac{\langle \sigma v \rangle dN_\gamma}{4\pi m_{dm}^2 dE_\gamma} \times \int_0^\infty \rho^2(r) dl
\]

- *Astrophysical factor* suggests looking at GC, dwarf spheroidals, . . .
- Photons and neutrinos point back to the source, while charged particles diffuse.
The annihilation cross-section

Annihilations in the halo are non-relativistic, \( \nu \approx 10^{-3} \).
The amplitude is analytical for \( k \to 0 \)

\[
\mathcal{M} \propto \int e^{ikx} V_{\text{Born}}(x)
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Including factors of \( k^l Y^m_l \) in a partial wave expansion, \( \sigma \propto k^{2l-1} \)

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\sigma \nu = a + bv^2 + \ldots
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More complicated velocity dependence

If there are new light particles mediating long-range forces between the dark matter, an enhancement occurs at low velocities:

\[ \sigma \to \sigma \times \frac{\pi \alpha}{v} \]

If annihilation proceeds near a resonant state,

\[ \nu \sigma \propto \frac{1}{(v^2/4 + \Delta)^2 + \Gamma_A^2 (1 - \Delta)/4m_\chi^2} \]

Enhancements at low velocities, \( \nu \sim 10^{-3} \), different than at decoupling.
More complicated velocity dependence

If there are new light particles mediating long-range forces between the dark matter, an enhancement occurs at low velocities:

$$\sigma \rightarrow \sigma \times \frac{\pi \alpha}{v}$$

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$$v\sigma \propto \frac{1}{(v^2/4 + \Delta)^2 + \Gamma_A^2 (1 - \Delta)/4m^2_\chi}$$

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Substructure enhanced

Lattanzi & Silk
Substructure enhanced

Lattanzi & Silk
What went in calculating the flux?

The averaged cross-section

\[ \langle \sigma v \rangle \rightarrow S(v) \langle \sigma v \rangle \]

But, the flux is

\[ \Phi = \text{Rate} \times v_{\text{rel}} \]

We have to average this, using the dark matter velocity distribution.
What went in calculating the flux?

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But, the flux is

\[ \Phi = Rate \times v_{rel} \]

We have to average this, using the dark matter velocity distribution.
How should we calculate the fluxes?

\[ Flux \propto \int dv_{rel} dl_{los} f_{pair}(r, v_{rel}) \times \sigma v_{rel} \]

The usual approach assumes

\[ f_{pair}(r, v_{rel}) = \rho^2(r) \times f_{MB}(v_{rel}) \]

See Robertson & Zentner for an approximate Jeans' based analysis.
How should we calculate the fluxes?

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\text{Flux} \propto \int d\nu_{\text{rel}} d\ell_{\text{los}} f_{\text{pair}}(r, \nu_{\text{rel}}) \times \sigma \nu_{\text{rel}}
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Which velocity distribution?

Since DM is assumed to be heavy, use Maxwell-Boltzmann?

Kuhlen et al.
Which velocity distribution?

Since DM is assumed to be heavy, use Maxwell-Boltzmann?

Kuhlen et al.
Obtaining the phase-space distribution

Assume that dark matter satisfies the collisionless Boltzmann equation,

\[ \frac{df}{dt} = 0 \]

Very hard to solve! Only a few exact solutions known, found finding integrals of motion (
\textit{singular isothermal sphere}, Hernquist, Jaffe, . . . ).

Taking velocity moments we obtain the Jeans’ equation:

\[ v_c^2 = \frac{GM(r)}{r} = -\bar{v}_r^2 \left( \frac{d \log \nu}{d \log r} + \frac{d \log \bar{v}_r^2}{d \log r} + 2\beta \right). \]

Necessary condition, useful to obtain density profiles from observational data.
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Necessary condition, useful to obtain density profiles from observational data.
Eddington’s formula

Gives the phase space distribution, if we know the density profile:

\[
 f(\mathcal{E}) = \frac{1}{\sqrt{8\pi^2}} \int_{0}^{\mathcal{E}} \frac{d\Psi}{\sqrt{\mathcal{E} - \Psi}} \frac{d^2\rho}{d\psi^2}.
\]  

(6)

Given a profile (NFW, cored, . . . ) we obtain the phase space distribution, which provides a full description of the dark matter distribution.

Check that \( \rho(r) \equiv \int d^3v f(\mathcal{E}) \).
Eddington’s formula

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Check that \( \rho(r) \equiv \int d^3v \, f(\mathcal{E}) \).
The phase-space distribution

\[
\log(\tilde{f}/g(c))
\]

- NFW
- Einasto
- NFW + baryons
Deriving the velocity distribution

Now that we have the full distribution function, we can find the velocity distribution at each point:

\[ P(v) = \frac{f(\psi - v^2/2)}{\rho(\psi)} \]

Check that \( \int P(v) dv = 1. \)
Deriving the velocity distribution

Now that we have the full distribution function, we can find the velocity distribution at each point:

\[ P(v) = \frac{f \left( \psi - \frac{v^2}{2} \right)}{\rho(\psi)} \]

Check that \( \int P(v) dv = 1. \)
Velocity distribution

\[ P_r(v) \text{ [(km/s)]}^{-1} \]

- 0.1 kpc
- 1 kpc
- 10 kpc

\[ v \text{ [km/s]} \]

0 100 200 300 400 500
Obtaining the relative velocity distribution

We move to the CM, to obtain $P_{rel}(v_{rel})$:

$$f_{sp}(\vec{v}_1)f_{sp}(\vec{v}_2)d\vec{v}_1d\vec{v}_2 = f_{pair}(\vec{v}_{cm}, \vec{v}_{rel})d\vec{v}_{cm}d\vec{v}_{rel}. \quad (7)$$

$$f_{rv}(v_{rel}) = 4\pi v_{rel}^2 2\pi \int_{0}^{\infty} dv_{cm} v_{cm}^2 \int_{0}^{\pi} d\theta \sin(\theta)$$

$$\cdot f_{sp}\left(\sqrt{v_{rel}^2/4 + v_{cm}^2 + v_{rel}v_{cm}\cos(\theta)}\right)$$

$$\cdot f_{sp}\left(\sqrt{v_{rel}^2/4 + v_{cm}^2 - v_{rel}v_{cm}\cos(\theta)}\right). \quad (8)$$

Recover the standard results for a Maxwell-Boltzmann distribution.
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$$f_{rv}(v_{rel}) = 4\pi v_{rel}^2 2\pi \int_0^\infty d\nu_{cm} \nu_{cm}^2 \int_0^\pi d\theta \sin(\theta) \cdot f_{sp} \left( \frac{v_{rel}^2}{4} + \nu_{cm}^2 + v_{rel} \nu_{cm} \cos(\theta) \right) \cdot f_{sp} \left( \frac{v_{rel}^2}{4} + \nu_{cm}^2 - v_{rel} \nu_{cm} \cos(\theta) \right). \quad (8)$$

Recover the standard results for a Maxwell-Boltzmann distribution.
Relative velocity distribution

\[ P_{\text{rel}}(v_{\text{rel}}) \left[ (\text{km/s})^{-1} \right] \]

- 0.1 kpc
- 1 kpc
- 10 kpc
We have all the ingredients to perform the los integration:

\[
\text{Flux} \propto \int dv_{\text{rel}} dl_{\text{los}} f_{\text{pair}}(r, v_{\text{rel}}) \times \sigma v_{\text{rel}}
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Or a volume integration, if we are interested in \( e^\pm \) yields in the center of the galaxy.
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Boost factor

Enhancements up to 1000!
Conclusions

- A full general relativistic treatment shows significant deviations of the DM distribution around a black hole. They could affect tests of no-hair theorems.
- Gamma-ray and neutrino fluxes might depend on the velocity distribution, which generically deviates from the naive Maxwell-Boltzmann approximation.
- Using the full phase space distribution from the Eddington inversion suggests that fluxes from the center of the halo are up to $10^3$ times larger.
- Constraints on Sommerfeld enhanced models from IC, synchroton or diffuse backgrounds might have to be re-evaluated.
- The velocity distribution also affects direct detection rates.